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Charles Lamar Phillips

AN INVESTIGATION OF N MAGNETICALLY-COUPLED VACUUM TUBE CIRCUITS

A THESIS

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AN INVESTIGATION OF N MAGNETICALLY-

COUPLED VACUUM TUBE CIRCUITS

Approved:

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I wish to express my sincere appreciation to Doctor D. L. Finn for his invaluable assistance in this research.

I wish to express deep appreciation to my wife, Laverne, for her encouragement and understanding.

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LIST OF SYMBOLS

a	Ratio of transformer primary turns to secondary turns.
e_c	Instantaneous total voltage rise from cathode to grid.
e_g	Instantaneous value of grid-signal voltage.
E_{cc}	Grid-circuit supply voltage rise from cathode.
e_b	Instantaneous total voltage rise from cathode to anode.
e_p	Instantaneous voltage across transformer primary coil.
E_{bb}	Anode-circuit supply voltage rise from cathode.
i_b	Instantaneous total anode current.
i_p	Instantaneous value of a-c component of anode current.
I_{bo}	Quiescent value of anode current.
N	Number of coil turns.
ϕ	Magnetic flux.
R_b	Value of load resistance as seen by each tube.
R_L	Actual load resistance.
u	Amplification factor of vacuum tube.

ABSTRACT

In this investigation four different types of transformer-coupled amplifiers are investigated. Each circuit is an amplifier composed of an arbitrary number of identical tubes. The four circuits differ in the method of applying the grid-signal voltages to the grids of the tubes, and in the topology of the magnetic circuits of the output transformers.

The circuits were investigated first assuming linear operation, and then assuming non-linear operation. The output transformers were assumed ideal in all cases. In the investigation of non-linear operation, composite characteristics for the circuits were constructed from static characteristics of the tubes. The circuits were then analyzed using these composite characteristics.

The first circuit to be investigated was the push-pull circuit in which the number of tubes was increased to an arbitrary even number, n, and the number of primary coils on the output transformer increased to the same number, n. The operation of this circuit is very similar to that of the simple push-pull amplifier, and the advantages are the same.

The second circuit to be investigated differed from the first circuit only in that the grid-signal

voltages were all in phase and certain primary windings of the transformer were reversed. This circuit offers no outstanding advantages.

The third circuit to be investigated was the same circuit as the first, except for the output transformer. For this circuit, the n primary coils and the secondary coil of the output transformer are magnetically in parallel. The advantages of this circuit were found to be principally the same as those of the push-pull amplifier. One additional advantage of this circuit is the theoretical cancellation of all a-c components of current in the power-supply current. One disadvantage of this circuit is the existence of d-c magnetization in the primary legs of the output transformer.

The fourth circuit to be investigated was different from the third circuit only in that the grid-signal voltages were all in phase and certain primary windings of the transformer were reversed. This circuit offers no outstanding advantages.

The operation of the third and fourth circuits was determined experimentally with the circuits composed of two tubes. The experimental operation was found to be substantially the same as the theoretical operation.

CHAPTER I

INTRODUCTION

History.--The transformer-coupled amplifier has found wide application in the field of electronics. When compared to the direct coupled amplifier, the transformer-coupled amplifier has some disadvantages. However, it also has many advantages. One of them is the reduction of non-linear distortion, which is obtained by the arrangement of the tube circuits and the magnetic circuit of the output transformer. The principal transformer-coupled circuit used for the reduction of non-linear distortion is the push-pull circuit.

The principal advantages (1)* of the push-pull circuit are:

1. reduction in non-linear distortion, making it possible to obtain increased power output (compared with single-sided operation) for a specified maximum distortion,
2. reduced sensitivity to ripple voltage in the power supply,
3. reduced current of signal frequency flowing

*Numbers in parentheses refer to the bibliography unless preceded by the word "equation."

through the source of power supply, and

4. reduced magnetization of the output-transformer core.

While the advantages of the push-pull circuit are many, some questions arise concerning it:

1. What is the effect of adding more vacuum tubes to the circuit and the corresponding primary coils to the output transformer?
2. What is the effect of putting all grid-signal voltages in phase under the conditions of question 1?
3. What are the effects of changing the topology of the magnetic circuit of the output transformer under the conditions of questions 1 and 2?

No answers could be found to these questions in the available literature. It was the purpose of this study to investigate these questions.

Problem.--The problem of this study is the investigation of four different types of vacuum tube circuits in which the outputs of the vacuum tubes are magnetically coupled. In each circuit, all tubes are assumed to be identical. The first of these circuits is the push-pull circuit in which the number of vacuum tubes has been increased to n and the output transformer has n corresponding primary coils. The number n will be even in all cases. Such a transformer is shown in Fig. 1. This circuit will be referred to as the push-pull series magnetic circuit. The second circuit to be investigated employs the same transformer as the first, but in this circuit all grid-signal voltages are in phase. It will be referred to as the common-input series magnetic circuit.

The third circuit to be investigated is a circuit of n vacuum tubes in which half the grid-signal voltages are 180° out of phase and the output transformer coils are magnetically in parallel. Such a transformer is shown in Fig. 2. This circuit will be referred to as the push-pull parallel magnetic circuit. The fourth circuit to be investigated employs the same transformer as the third, but in this circuit all grid-signal voltages are in phase. It will be referred to as the common-input parallel magnetic circuit.

The effect on circuit operation of increasing the number of tubes will be investigated for each type of circuit. The advantages and disadvantages of the different types of circuits will be compared. The operation of the circuits when composed of two tubes will be determined experimentally, and compared with the theoretical operation.

The first type of circuit mentioned could be considered as a generalized push-pull circuit, and the analysis of the push-pull circuit is found in almost any book on electronics. However, no mention could be found of the other three types of circuits in available literature.

CHAPTER II

IDEAL MAGNETIC COUPLING

Review.--In the analysis of the conventional push-pull circuit, the output transformer is assumed to be ideal for a-c operation. In this investigation, all magnetic coupling will be assumed ideal for a-c operation.

Ideal magnetic coupling is based on the following three postulates (2):

1. Each coil-voltage is directly proportional to the time rate of change of magnetic flux through the coil.
2. The sum of the ampere-turns around any closed magnetic path equals zero.
3. The sum of the magnetic flux entering each magnetic node equals zero.

The first postulate, stated in the form of an equation, is

$$v = -N \frac{d\phi}{dt} \tag{1}$$

where v is the voltage across the coil, N is the number of turns in the coil, and $\frac{d\phi}{dt}$ is the time rate of change of magnetic flux through the coil. This postulate implies that no resistance or capacitance is associated with the coil.

The second postulate, stated in the form of an equation, is

$$\sum Ni = 0 \tag{2}$$

for any closed magnetic path. In equation (2) the directions of the magnetomotive forces must be taken into consideration. This postulate implies that no reluctance is associated with the magnetic paths.

The third postulate, stated in the form of an equation, is

$$\sum \phi = 0 \tag{3}$$

for any magnetic node. In equation (3) the directions of the magnetic flux must be taken into consideration. This postulate implies that all the magnetic flux is confined to the core structure.

Solution of Two Magnetic Circuits.--The operation of a magnetic circuit can be described by use of equations (1), (2), and (3). With the use of these equations and the equations of the electrical circuits connected between coil terminals, the operation of the magnetically coupled electrical circuits can be described.

The magnetic circuits of two transformers will now be analyzed. These two transformers are to be used in the vacuum tube circuits to be investigated. The first of these transformers is shown in Fig. 1. It will be

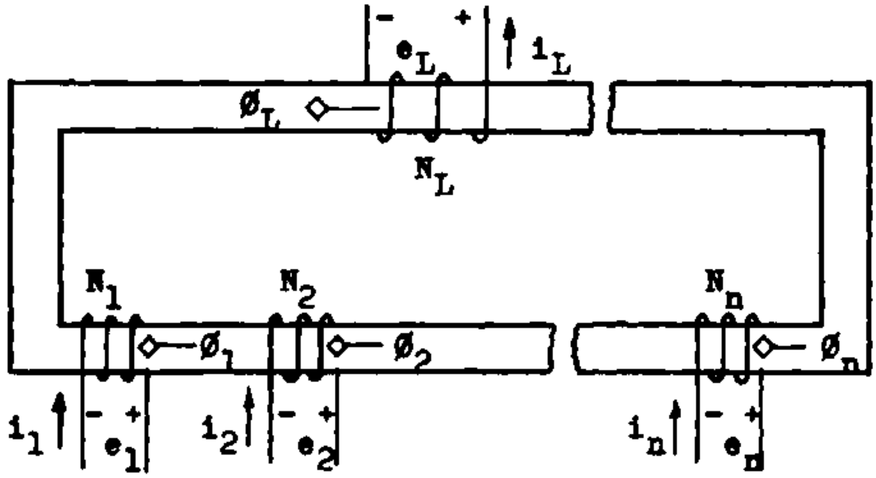


Fig. 1. Transformer With Series Magnetic Circuit.

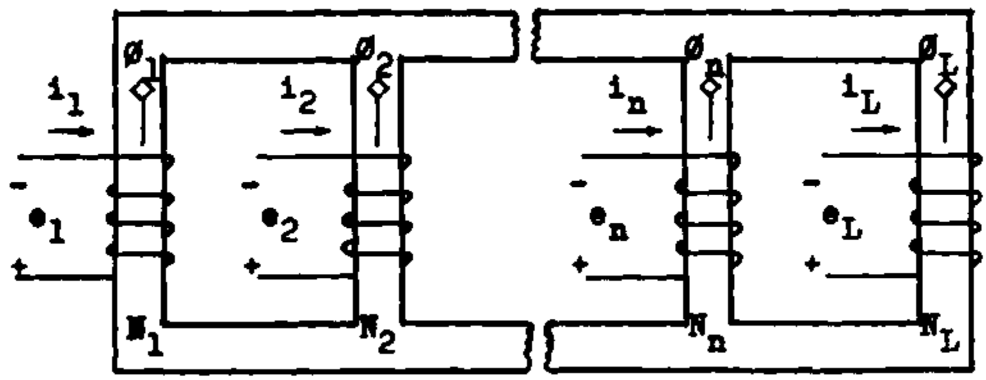


Fig. 2 Transformer With Parallel Magnetic Circuit.

noted that the coils are magnetically in series.

The sum of the magnetic flux into a node equals zero. Consider a point between each two adjacent coils as the magnetic nodes. Then

$$\begin{aligned} \phi_1 - \phi_2 &= 0 \\ \phi_2 - \phi_3 &= 0 \\ \cdots&\cdots \\ \phi_n + \phi_L &= 0 \end{aligned} \tag{4}$$

Or

$$\phi_1 = \phi_2 = \phi_3 = \ldots = \phi_n = -\phi_L \tag{5}$$

Differentiating with respect to time,

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} = \frac{d\phi_3}{dt} = \ldots = \frac{d\phi_n}{dt} = -\frac{d\phi_L}{dt} \tag{6}$$

From equation (1),

$$\frac{d\phi_1}{dt} = -\frac{1}{N_1}\left[-N_1 \frac{d\phi_1}{dt}\right] = -\frac{e_1}{N_1} \tag{7}$$

Therefore

$$\frac{e_1}{N_1} = \frac{e_2}{N_2} = \frac{e_3}{N_3} = \ldots = \frac{e_n}{N_n} = -\frac{e_L}{N_L} \tag{8}$$

The sum of the magnetomotive forces around any closed magnetic path equals zero. For Fig. 1,

$$N_1 i_1 + N_2 i_2 + \ldots + N_n i_n - N_L i_L = 0 \tag{9}$$

The operation of the transformer of Fig. 1 is described by equations (8) and (9). Equations (8) relate the voltages of the coils to each other, and equation (9) relates the currents through the coils to each other.

The transformer of Fig. 2 will now be analyzed. In this transformer the coils are magnetically in parallel. The sum of the flux into a magnetic node equals zero. Consider the top node:

$$\phi_1 + \phi_2 + \ldots + \phi_n + \phi_L = 0 \tag{10}$$

Differentiating with respect to time,

$$\frac{d\phi_1}{dt} + \frac{d\phi_2}{dt} + \ldots + \frac{d\phi_n}{dt} + \frac{d\phi_L}{dt} = 0 \tag{11}$$

Using equation (7), equation (11) becomes

$$\frac{e_1}{N_1} + \frac{e_2}{N_2} + \ldots + \frac{e_n}{N_n} + \frac{e_L}{N_L} = 0 \tag{12}$$

The sum of the magnetomotive forces around a closed path equals zero. From Fig. 2,

$$\begin{aligned} N_1 i_1 - N_2 i_2 &= 0 \\ N_2 i_2 - N_3 i_3 &= 0 \\ \cdots\cdots\cdots \\ N_n i_n - N_L i_L &= 0 \end{aligned} \tag{13}$$

Or

$$N_1 i_1 = N_2 i_2 = \ldots N_n i_n = N_L i_L \tag{14}$$

The operation of the transformer of Fig. 2 is described by equations (12) and (14). Equation (12) relates the voltages of the coils to each other, and equations (14) relate the currents through the coils to each other.

CHAPTER III

PUSH-PULL SERIES MAGNETIC CIRCUIT

Linear Analysis.--The first circuit to be investigated is the circuit of Fig. 3. It will be noted that the primary coils of the output transformer are magnetically in series, and that the polarity of the output coils of the even-numbered tubes is opposite to that of the odd-numbered tubes. Because of this reversal in polarity, the equations of the transformer will be changed. Assume the number of turns of the primary coils to be equal, and let a be the ratio of the turns of one primary coil to the turns of the secondary coil. Then the equations of the transformer are, considering equations (8) and (9),

$$e_L = -\frac{1}{a} e_{p_1} = \frac{1}{a} e_{p_2} = \dots = \frac{1}{a} e_{p_n} \qquad (15)$$

$$i_L = a(i_{p_1} - i_{p_2} + \dots + i_{p_{n-1}} - i_{p_n}) \qquad (16)$$

If the operation of the vacuum tubes is considered to be in the linear regions of the static characteristic, then Fig. 4 is the a-c equivalent circuit of Fig. 2. It will be noted that all grid-signal voltages are equal, and that the grid-signal voltage of the even-numbered tubes is 180° out of phase with that of the odd-numbered tubes. For the circuit of Fig. 2, n+1 loop equations can

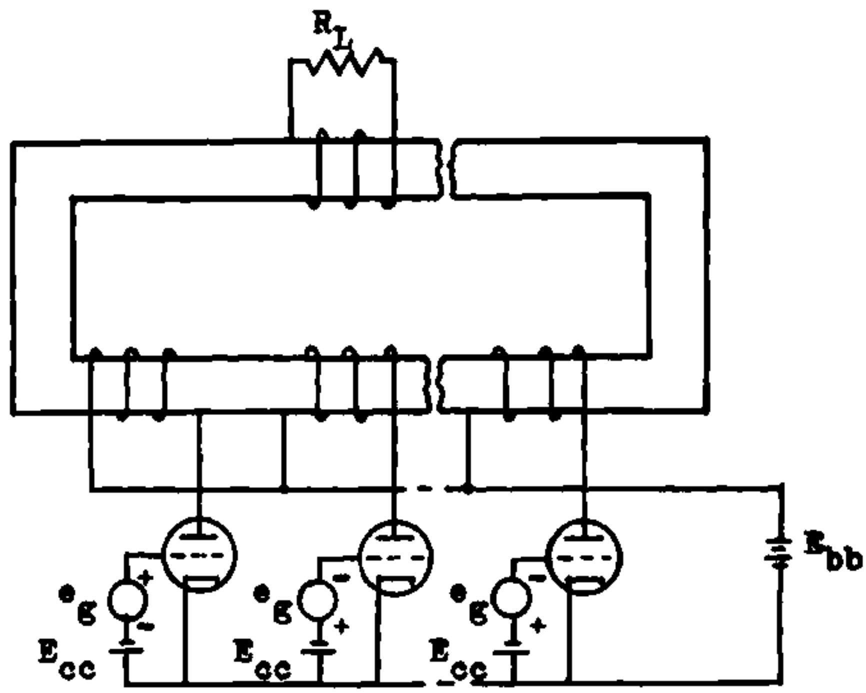


Fig. 3. Push-Pull Series Magnetic Circuit.

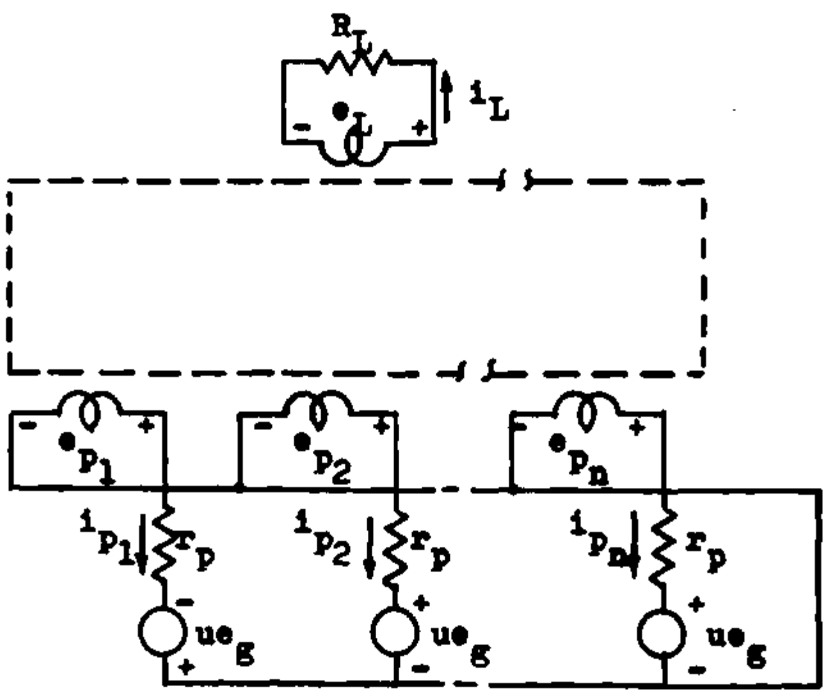


Fig. 4. Equivalent A-C Circuit of Push-Pull Series Magnetic Circuit.

be written for the electrical circuits. Consider the n equations obtained by writing a loop equation for each tube circuit.

$$
\begin{aligned}
ue_g - i_{p_1}r_p + e_{p_1} &= 0 \\
ue_g + i_{p_2}r_p - e_{p_2} &= 0 \\
\cdots\cdots\cdots\cdots\cdots \\
ue_g + i_{p_n}r_p - e_{p_n} &= 0
\end{aligned} \tag{17}
$$

Adding these equations,

$$
\begin{gathered}
nue_g - r_p(i_{p_1} - i_{p_2} + \ldots - i_{p_n}) + \\
(e_{p_1} - e_{p_2} + \ldots - e_{p_n}) = 0
\end{gathered} \tag{18}
$$

Substituting equations (15) and (16) into (18),

$$nue_g - \frac{i_L}{a}(r_p) - nae_L = 0 \tag{19}$$

And

$$e_L = i_L R_L \tag{20}$$

Therefore equation (19) becomes

$$nue_g - \frac{i_L}{a}(r_p) - nai_L R_L = 0 \tag{21}$$

Solving for i_L,

$$i_L = \frac{nue_g}{(\frac{1}{a})r_p + naR_L} \tag{22}$$

and

$$e_L = \frac{nue_gR_L}{(\frac{1}{a})r_p + naR_L} \tag{23}$$

The gain A is

$$A = \frac{e_L}{e_g} = \frac{nuR_L}{(\frac{1}{a})r_p + naR_L} \tag{24}$$

From equation (15),

$$e_{p_1} = -e_{p_2} = e_{p_3} = \dots = -e_{p_n} \tag{25}$$

Call the value of the individual terms of equations (25) e_p. Then, from equations (15) and (23),

$$e_p = -ae_L = -\frac{nue_gR_L}{r_{p(\frac{1}{a})}^2 + nR_L} \tag{26}$$

The plate voltage of the tubes is given by

$$e_b = E_{bb} + e_p \tag{27}$$

Thus the plate voltage of the even-numbered tubes is

$$e_b = E_{bb} - \frac{nue_gR_L}{r_{p(\frac{1}{a})}^2 + nR_L} \tag{28}$$

and the plate voltage of the odd-numbered tubes is

$$e_b = E_{bb} + \frac{nue_gR_L}{r_{p(\frac{1}{a})}^2 + nR_L} \tag{29}$$

From equations (17) and (25),

$$i_{p_1} = - i_{p_2} = i_{p_3} = \ldots = - i_{p_n} \tag{30}$$

Call the value of the individual terms of equations (30) i_p. Then equation (16) becomes

$$i_L = ani_p \tag{31}$$

Thus the plate current of the even-numbered tubes is, from equations (22) and (31),

$$i_p = \frac{i_L}{an} = \frac{ue_g}{r_p + a^2nR_L} \tag{32}$$

The plate current of the odd-numbered tubes is the negative of equation (32). From equation (32), the value of load resistance as seen by each tube is

$$R_b = a^2nR_L \tag{33}$$

<u>Non-Linear Analysis</u>.--An analysis of the circuit of Fig. 3 will now be made assuming large signal (non-linear) operation. With the vacuum tubes operating in the non-linear regions of their static characteristic, the equivalent circuit of Fig. 4 is no longer valid. A graphical analysis will be made using the static characteristics of the tubes.

The a-c operation of the transformer is not affected by the region of operation of the tubes; therefore the transformer equations will be unchanged. These equations are equations (15) and (16).

The following set of equations is obtained from Fig. 3.

$$e_{b_1} = E_{bb} + e_{p_1} \tag{34}$$
$$e_{b_2} = E_{bb} + e_{p_2}$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$e_{b_n} = E_{bb} + e_{p_n}$$

where e_b is the plate voltage of the vacuum tube in question. From equation (15),

$$e_{p_1} = e_{p_3} = \cdots = e_{p_{n-1}} \tag{35}$$

$$e_{p_2} = e_{p_4} = \cdots = e_{p_n} \tag{36}$$

Therefore

$$e_{b_1} = e_{b_3} = \cdots = e_{b_{n-1}} \tag{37}$$

$$e_{b_2} = e_{b_4} = \cdots = e_{b_n} \tag{38}$$

Call the value of the individual terms of equations (37) e_b, and the value of the individual terms of equations (38) e_b'.

Since the tubes are identical, their static characteristics are identical. An inspection of Fig. 3 will show that the quiescent operating points are the same. Thus the quiescent plate currents of the tubes will be equal. Or

$$I_{bo_1} = I_{bo_2} = \dots = I_{bo_n} \tag{39}$$

Call the value of the individual terms of equations (39) I_{bo}. Then, from Fig. 1,

$$\begin{aligned} i_{b_1} &= I_{bo} + i_{p_1} \\ i_{b_2} &= I_{bo} + i_{p_2} \\ &\cdots\cdots \\ i_{b_n} &= I_{bo} + i_{p_n} \end{aligned} \tag{40}$$

Also,

$$\begin{aligned} e_{c_1} &= E_{cc} - e_g \\ e_{c_2} &= E_{cc} + e_g \\ &\cdots\cdots \\ e_{c_n} &= E_{cc} + e_g \end{aligned} \tag{41}$$

where e_c is the cathode-to-grid voltage of the tube in question.

Thus, at any instant of time, the grid voltages of the even-numbered tubes are equal, and the grid voltages of the odd-numbered tubes are equal. Also, from equations (37) and (38), at any instant of time the plate voltages of the even-numbered tubes are equal, and the plate voltages of the odd-numbered tubes are equal. Therefore, at any instant of time, the plate currents of the even-numbered tubes are equal, and the plate currents of the odd-numbered tubes are equal. Or

$$i_{b_1} = i_{b_3} = \dots = i_{b_{n-1}} \tag{42}$$

$$i_{b_2} = i_{b_4} = \dots = i_{b_n} \tag{43}$$

Call the value of the individual terms of equations (42) i_b, and of equations (43) i_b'. Then equations (40) become

$$\begin{aligned} i_{p_1} &= i_b - I_{bo} \\ i_{p_2} &= i_b' - I_{bo} \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ i_{p_n} &= i_b' - I_{bo} \end{aligned} \tag{44}$$

Substituting equations (44) into (16),

$$i_L = a\left(\frac{n}{2}\, i_b - \frac{n}{2}\, i_b'\right) = \frac{an}{2}(i_b - i_b') \tag{45}$$

From equations (15), (34), (35) and (36),

$$e_L = -\frac{1}{a}(e_b - E_{bb}) = \frac{1}{a}(e_b' - E_{bb}) \tag{46}$$

From equation (20)

$$i_L = \frac{E_L}{R_L} \tag{47}$$

Substituting equations (45) and (46) into (47),

$$\frac{an}{2}(i_b - i_b') = \frac{-\frac{1}{a}(e_b - E_{bb})}{R_L} \tag{48}$$

Or

$$e_b = E_{bb} - (a^2)\,\frac{n}{2}\,R_L(i_b - i_b') \tag{49}$$

and

$$e_b' = E_{bb} + (a^2)\,\frac{n}{2}\,R_L(i_b - i_b') \tag{50}$$

From equations (41),

$$e_{c_1} = e_{c_3} = \ldots = e_{c_{n-1}} = E_{cc} - e_g \tag{51}$$

$$e_{c_2} = e_{c_4} = \ldots = e_{c_n} = E_{cc} + e_g \tag{52}$$

Equations (49), (50), (51), and (52) describe the operation of the circuit of Fig. 3. Equations (49) and (50) relate the plate currents and the plate voltages of the vacuum tubes. These relationships can be used to construct a composite characteristic for the circuit. But equations (49) and (50) are the relationships of a two-tube push-pull amplifier (3), except for the factor of n/2. The factor n/2 is a multiplier of the current factor of the equations. Therefore, if the current scale of the two-tube push-pull composite characteristic is multiplied by a factor of n/2, then the resultant characteristic will be the desired composite characteristic of Fig. 3. It is felt that nothing will be added to this study by going through the steps necessary to construct a two-tube push-pull composite characteristic, or analyzing this composite characteristic. The method of construction and the analysis can be found in almost any good book on vacuum-tube amplifiers. However, it will be noted that there will be no d-c magnetization of the core of the transformer. This can be seen from the fact that the d-c magnetomotive force of half the windings acts in one direction, while that of the other half of the windings acts in the opposite direction. Thus the resultant d-c magnetomotive force is zero.

Consider equations (49) and (50). Suppose the factor n/2 is considered as being part of the load resistance. Then the composite characteristic will be identical with the composite characteristic of the two-tube push-pull circuit, but the load line will have a slope of

$$\text{slope} = \frac{1}{(\frac{n}{2})a^2R_L} \tag{53}$$

For the first composite characteristic mentioned, the current scale is expanded to allow for the factor n/2. For the composite characteristic of the two-tube push-pull circuit, the slope of the load line is contracted to allow for the factor of n/2. In either case the path of operation on the composite characteristic is the same.

The effect of increasing n will now be considered.

From equation (53), if n is increased, then the slope of the load line is decreased. Thus the value of $(i_b - i_b')$ will decrease for any instant of time. Consider the case where as the number of tubes in increased by n/2, the load resistance is made equal to $2R_L/n$. Then, from equation (53), the slope of the load line will not change as n is increased, and $(i_b - i_b')$ will not change. Also, from the linear analysis, equation (33), the value of load resistance as seen by each tube will remain constant if the actual load resistance is made equal to $2R_L/n$. The power output of the circuit is

$$P_L = i_L{}^2 R_L \qquad (54)$$

From equations (45) and (54),

$$P_L = \frac{a^2n^2}{4}(i_b - i_b')^2 R_L = \frac{a^2n}{2}(i_b - i_b')^2 R_1 \qquad (55)$$

where

$$R_1 = \frac{R_L}{\frac{n}{2}} \qquad (56)$$

Thus from equation (55), as the number of tubes is increased, the power output is increased by the same factor, provided that the load resistance is given by equation (56).

If R_L is held constant as the number of tubes is increased, then, for any instant of time, $(i_b - i_b')$ will decrease. From equation (55), the n^2 term will tend to increase the power out, but the $(i_b - i_b')^2$ term will tend to decrease the power out. The actual effect of increasing the number of tubes for constant R_L will therefore depend on the characteristics of the tubes used, the value of R_L, and the number n. However, it will be noted that if the number of tubes is increased, the circuit will be capable of a higher power output, because of the increased number of tubes.

From the linear analysis, equation (24), if the number of tubes is increased, the voltage gain of the circuit will not be affected provided the load resistance is given by equation (56). However, from equation (24), if the load resistance is held constant as the number of tubes is increased, the voltage gain of the circuit will increase by a factor less than the factor by which the number of tubes is increased. This increase is determined by the values of a, r_p and R_L.

It will be noted from equations (16), (49), (50), and (51) that the effect of the transformer is to put n/2 two-tube push-pull circuits in parallel. That is, except for the factor of the turns ratio, the load current is the sum of the currents of n/2 two-tube push-pull circuits.

CHAPTER IV

COMMON-INPUT SERIES MAGNETIC CIRCUIT

Linear Analysis.--The second circuit to be investigated is the circuit of Fig. 5. It will be noted that the primary coils of the output transformer are magnetically in series, and that the polarity of all primary coils is the same. Assume the number of turns of the primary coils to be equal, and let a be the ratio of the turns of one primary coil to the turns of the secondary coil. Then the equations of the transformer are, from equations (8) and (9),

$$e_L = -\frac{1}{a}\,e_{p_1} = -\frac{1}{a}\,e_{p_2} = \dots = -\frac{1}{a}\,e_{p_n} \qquad (57)$$

$$i_L = a(i_{p_1} + i_{p_2} + \dots + i_{p_n}) \qquad (58)$$

If the operation of the vacuum tubes is considered to be in the linear regions of their static characteristic, then Fig. 6 is the a-c equivalent circuit of Fig. 5. It will be noted that the grid-signal voltages of the vacuum tubes are equal and in phase. For this circuit, n+1 loop equations can be written. Consider the n equations obtained by writing a loop equation for each tube circuit.

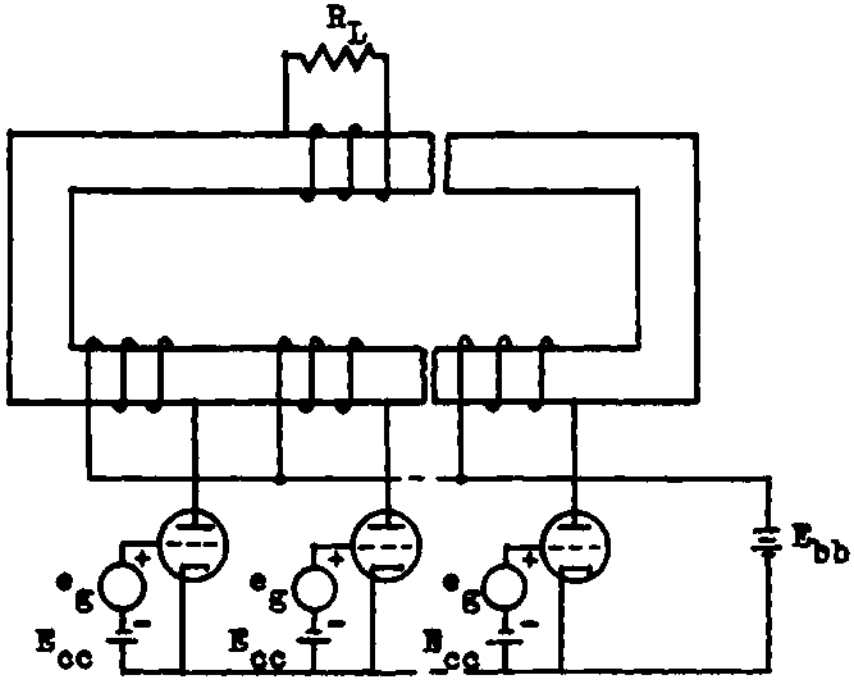


Fig. 5. Common-Input Series Magnetic Circuit.

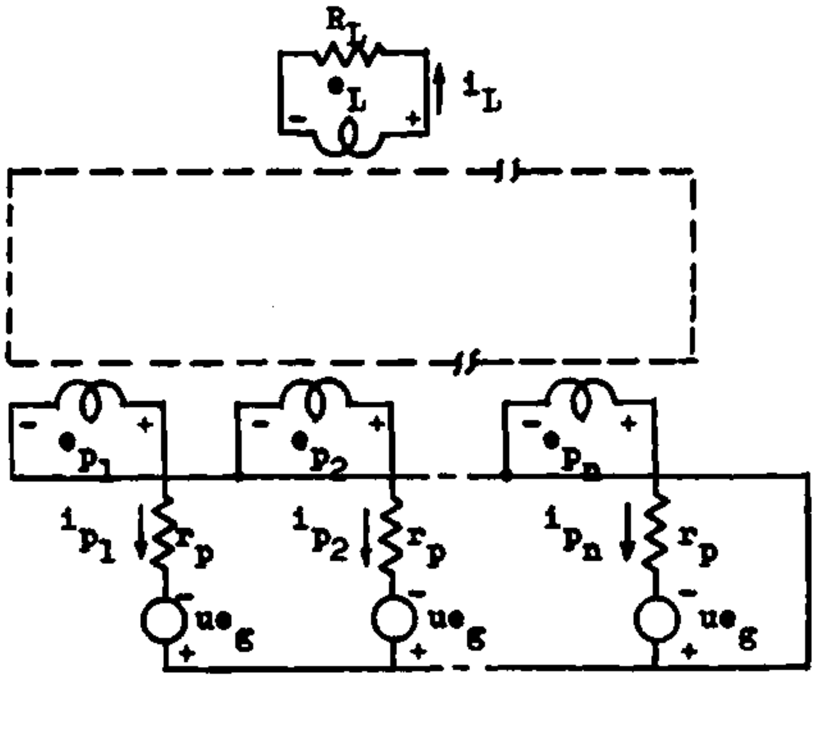


Fig. 6. Equivalent A-C Circuit of Common-Input Series Magnetic Circuit.

$$ue_g - i_{p_1}r_p + e_{p_1} = 0 \tag{59}$$

$$ue_g - i_{p_2}r_p + e_{p_2} = 0$$

$$\cdots\cdots\cdots\cdots$$

$$ue_g - i_{p_n}r_p + e_{p_n} = 0$$

Adding these equations

$$nue_g - r_p(i_{p_1} + i_{p_2} + \ldots + i_{p_n}) + (e_{p_1} + e_{p_2} + \ldots + e_{p_n}) = 0 \tag{60}$$

Substituting equations (57) and (58) into (60),

$$nue_g - (\tfrac{1}{a})r_p i_L - ane_L = 0 \tag{61}$$

Substituting equation (20) into (61),

$$nue_g - (\tfrac{1}{a})r_p i_L - ani_L R_L = 0 \tag{62}$$

Solving for i_L,

$$i_L = \frac{nue_g}{(\frac{1}{a})r_p + anR_L} \tag{63}$$

and from equation (20),

$$e_L = \frac{nue_g R_L}{(\frac{1}{a})r_p + anR_L} \tag{64}$$

The gain A is

$$A = \frac{e_L}{e_g} = \frac{nuR_L}{(\frac{1}{a})r_p + anR_L} \qquad (65)$$

From equation (57),

$$e_{p_1} = e_{p_2} = \ldots = e_{p_n} \qquad (66)$$

Call the value of the individual terms of equation (66) e_p. Then, from equations (57) and (64),

$$e_p = -ae_L = \frac{-\,nue_g}{r_p(\frac{1}{a}2) + nR_L} \qquad (67)$$

The plate voltage of the tubes is

$$e_b = E_{bb} + e_p \qquad (68)$$

Thus

$$e_b = E_{bb} - \frac{nue_g}{r_p(\frac{1}{a}2) + nR_L} \qquad (69)$$

From equations (59) and (66),

$$i_{p_1} = i_{p_2} = \ldots = i_{p_n} \qquad (70)$$

Call the value of the individual terms of equations (70) i_p. Then equation (58) becomes

$$i_L = ani_p \qquad (71)$$

Substituting equation (63) into (71),

$$i_p = \frac{i_L}{an} = \frac{ue_g}{r_p + a^2nR_L} \tag{72}$$

From equation (72), the value of load resistance as seen by each vacuum tube is

$$R_b = a^2nR_L \tag{73}$$

Non-Linear Analysis.--An analysis of the circuit of Fig. 5 will now be made, assuming large signal operation. With the tubes operating in the non-linear regions of their static characteristic, the equivalent circuit of Fig. 6 is no longer valid. A graphical analysis will be made using the static characteristics of the tubes.

The a-c operation of the transformer is not affected by the region of operation of the vacuum tubes; therefore the transformer equations will be unchanged. These equations are equations (57) and (58).

The following equations are obtained from Fig. 5.

$$\begin{aligned} e_{b_1} &= E_{bb} + e_{p_1} \\ e_{b_2} &= E_{bb} + e_{p_2} \\ &\cdots\cdots\cdots \\ e_{b_n} &= E_{bb} + e_{p_n} \end{aligned} \tag{74}$$

From equation (57),

$$e_{p_1} = e_{p_2} = \cdots = e_{p_n} \tag{75}$$

Call the value of the individual terms of equations (75) e_p. Thus, from equations (74) and (75),

$$e_{b_1} = e_{b_2} = \cdots = e_{b_n} \tag{76}$$

Call the value of the individual terms of equations (76) e_b. Thus, for any tube,

$$e_b = E_{bb} + e_p \tag{77}$$

Since the tubes are identical, their static characteristics are identical. An inspection of Fig. 5 will show that the quiescent operating points of the tubes are the same. Thus the quiescent plate currents of the tubes will be equal. Or

$$I_{bo_1} = I_{bo_2} = \cdots = I_{bo_n} \tag{78}$$

Call the value of the individual terms of equations (78) I_{bo}. Then, from Fig. 5,

$$\begin{aligned} i_{b_1} &= I_{bo} + i_{p_1} \\ i_{b_2} &= I_{bo} + i_{p_2} \\ &\cdots\cdots\cdots \\ i_{b_n} &= I_{bo} + i_{p_n} \end{aligned} \tag{79}$$

Also

$$e_{c_1} = E_{cc} - e_g \tag{80}$$
$$e_{c_2} = E_{cc} - e_g$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$e_{c_n} = E_{cc} - e_g$$

From equations (80), at any instant of time the grid voltages of the tubes are equal. And from equations (76), at any instant of time the plate voltages of the tubes are equal. Therefore, at any instant of time, the plate currents of the tubes are equal. Or

$$i_{b_1} = i_{b_2} = \cdots = i_{b_n} \tag{81}$$

Call the value of the individual terms of equations (81) i_b. Thus, from equations (79) and (81),

$$i_{p_1} = i_{p_2} = \cdots = i_{p_n} \tag{82}$$

Call the value of the individual terms of equations (82) i_p. Then, for any tube,

$$i_b = I_{bo} + i_p \tag{83}$$

From Fig. 5,

$$e_L = i_L R_L \tag{84}$$

From equations (58), (82), and (84),

$$e_L = i_L R_L = a n i_p R_L \tag{85}$$

Also, from equations (57), (77), and (85),

$$e_L = -\frac{1}{a}(e_b - E_{bb}) = ani_pR_L \tag{86}$$

Or, from equations (83) and (86),

$$ni_p = n(i_b - I_{bo}) = -\frac{1}{a^2R_L}(e_b - E_{bb}) \tag{87}$$

And from equations (80),

$$e_{c_1} = e_{c_2} = \dots = e_{c_n} = E_{cc} - e_g \tag{88}$$

Equations (87) and (88) describe the operation of the circuit of Fig. 5. Equation (87) is the relationship between tube currents and tube voltages to be used to construct a composite characteristic for the circuit. However, it will be noted that equation (87) is the relationship for the operation of one of the tubes of the circuit in which the current has been multiplied by a factor of n. Thus the composite characteristic for the circuit will be the static characteristic of the tubes with the current scale multiplied by a factor of n.

It will be noted from equation (87) that the path of operation is a straight line with a slope of

$$\text{slope} = -\frac{1}{a^2R_L} \tag{89}$$

on the composite characteristic. When no signal is applied to the circuit, $i_b = I_{bo}$. Since the transformer is assumed ideal, there is no resistance associated with the windings. Thus $e_b = E_{bb}$. Since the value of E_{cc} is known, then the value of nI_{bo} can be found from the composite characteristic of the circuit. The value of nI_{bo} will be the value of i_b at $e_b = E_{bb}$ and $e_c = E_{cc}$. This point is a point of the path of operation. Thus, with one point and the slope of the path of operation known, the load line can be constructed on the composite characteristic.

The composite characteristic for a circuit of 6J5 vacuum tubes is shown in Fig. 7. The path of operation is also shown for a value of R_L.

Equation (87) can also be written

$$i_p = - \frac{1}{na^2R_L} (e_b - E_{bb}) \qquad (90)$$

Thus the composite characteristic can also be represented by the static characteristic of the tubes with the original voltage and current scales, but the slope of the load line will then be

$$\text{slope} = - \frac{1}{na^2R_L} \qquad (91)$$

This composite characteristic is the equivalent of the first composite characteristic. In the first composite characteristic, the current scale is expanded to allow for the factor n. In the second composite characteristic

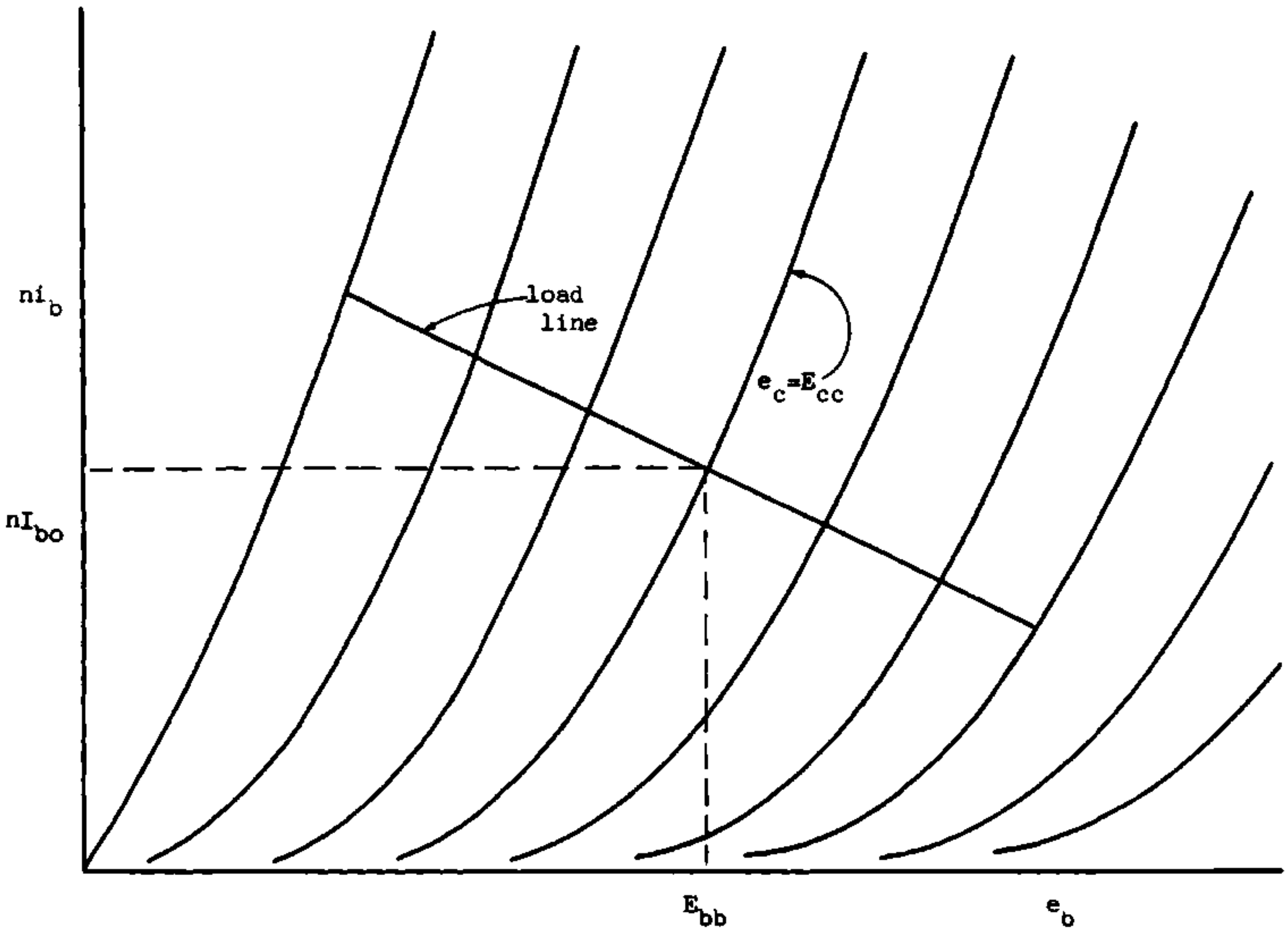


Fig. 7.--Composite Characteristic for Common-Input Series Magnetic Circuit.

the slope of the load line is contracted to allow for the factor n. In either case the path of operation on the composite characteristic is the same.

The treatment of the circuit of Fig. 5 can be made analogous to that of a single-tube circuit incorporating a "composite tube". In Fig. 5, if all the tubes except one are removed from the circuit, leaving the circuit open at the points where the tubes are removed, and if the plate current of the remaining tube is multiplied by a factor of n, then the remaining tube will be the composite tube. This circuit will satisfy the equations of the original circuit. These equations are equations (87) and (88).

The effect of increasing n will now be considered. From equation (91), if n is increased, then the slope of the load line is decreased. Thus the value of i_p will decrease. Consider the case where as the number of tubes is increased by n, the load resistance is made equal to R_L/n. Then, from equation (91), the slope of the load line will not change, and i_p will not change. Also, from the linear analysis, equation (73), the value of load resistance as seen by each tube will remain constant if the actual load resistance is made equal to R_L/n. The power output of the circuit is

$$P_L = e_L i_L \tag{92}$$

From equations (85) and (92),

$$P_L = n^2 i_p{}^2 a^2 R_L = n i_p{}^2 a^2 R_l \tag{93}$$

where

$$R_l = \frac{R_L}{n} \tag{94}$$

Thus from equation (93), as the number of tubes is increased, the power output is increased by the same factor, provided that the load resistance is given by equation (94). If R_L remains constant as the number of tubes is increased, the i_p will decrease. Thus, from equation (93), the n^2 term will tend to increase the power out, but the $i_p{}^2$ term will tend to decrease the power out. The actual effect of increasing the number of tubes for constant R_L will therefore depend on the characteristics of the tubes used, the value of R_L, and the number n. However, it will be noted that if the number of tubes is increased, the circuit will be capable of a higher power output, because of the increased number of tubes.

From the linear analysis, equation (65), if the number of tubes is increased, the voltage gain of the circuit will not be affected provided the load resistance is given by equation (94). However, from equation (65), if the load resistance is held constant as the number of tubes is increased, the voltage gain of the circuit will increase by a factor less than the factor by which the number of tubes is increased. This increase is determined by the values of a, r_p and R_L.

It will be noted from equations (57) and (58) that the effect of the transformer is to put the vacuum tubes in parallel with the load. That is, except for the factor of the turns ratio, the load current is the sum of the tube currents, and the load voltage is the same as the output voltage of each tube.

There will be no cancellation of harmonics in this circuit. Thus the distortion in the output will be comparable to that of a single tube amplifier. Also there will be no cancellation of harmonics in the power supply, since the currents add in the power supply. There will be a high degree of d-c magnetization of the core, since the d-c magnetomotive forces of all the primary windings act in the same direction.

CHAPTER V

PUSH-PULL PARALLEL MAGNETIC CIRCUIT

Linear Analysis.--The third circuit to be investigated is the circuit of Fig. 8. It will be noted that the primary coils of the output transformer are magnetically in parallel, and that the polarity of the output coils of the even-numbered tubes is opposite to that of the odd-numbered tubes. Because of this reversal in polarity, the equations of the transformer will be changed. Assume the primary coils have an equal number of turns, and let a be the ratio of the turns of one primary coil to the turns of the secondary coil. Then the equations of the transformer are, considering equations (12) and (14),

$$e_L = \frac{1}{a}(e_{p_2} - e_{p_1} + \dots + e_{p_n} - e_{p_{n-1}}) \qquad (95)$$

$$i_L = ai_{p_1} = -ai_{p_2} = \dots = -ai_{p_n} \qquad (96)$$

If the operation of the vacuum tubes is considered to be in the linear regions of the static characteristic, then Fig. 9 is the a-c equivalent circuit of Fig. 8. It will be noted that all the grid-signal voltages are equal, and that the grid-signal voltage of the even-numbered tubes is 180° out of phase with that of the odd-numbered tubes. For the circuit of Fig. 9, n+1 loop equations can

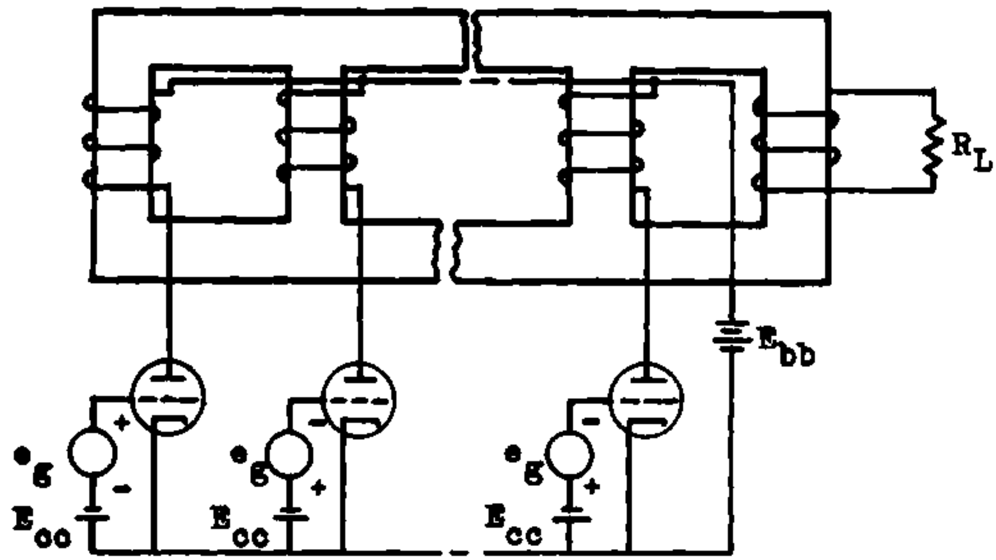


Fig. 8. Push-Pull Parallel Magnetic Circuit.

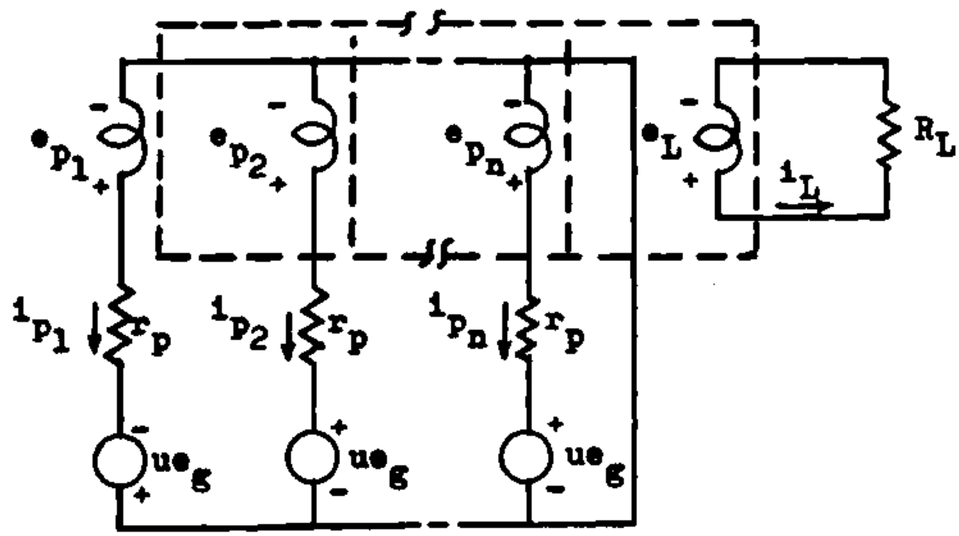


Fig. 9. Equivalent A-C Circuit of Push-Pull Parallel Magnetic Circuit.

be written for the electrical circuits. Consider the n equations obtained by writing a loop equation for each tube circuit.

$$
\begin{aligned}
ue_g - i_{p_1} r_p + e_{p_1} &= 0 \\
ue_g + i_{p_2} r_p - e_{p_2} &= 0 \\
\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot&\ \cdot\ \cdot\ \cdot\ \cdot \\
ue_g + i_{p_n} r_p - e_{p_n} &= 0
\end{aligned}
\tag{97}
$$

Adding these equations,

$$
\begin{gathered}
nue_g - r_p(i_{p_1} - i_{p_2} + \ldots - i_{p_n}) + \\
(e_{p_1} - e_{p_2} + \ldots - e_{p_n}) = 0
\end{gathered}
\tag{98}
$$

From equations (96),

$$i_{p_1} = - i_{p_2} = \ldots = - i_{p_n} \tag{99}$$

Call the value of the individual terms of equations (99) i_p. Substituting equations (95) and (99) into (98),

$$nue_g - ni_p r_p - ae_L = 0 \tag{100}$$

And

$$e_L = i_L R_L \tag{101}$$

Substituting equation (101) into (100), and solving for i_L,

$$i_L = \frac{ue_g}{(\frac{1}{a})r_p + \frac{a}{n}R_L} \tag{102}$$

and

$$e_L = \frac{ue_gR_L}{(\frac{1}{a})r_p + \frac{a}{n}R_L} \tag{103}$$

The gain A is

$$A = \frac{e_L}{e_g} = \frac{uR_L}{(\frac{1}{a})r_p + \frac{a}{n}R_L} \tag{104}$$

From equations (97) and (99),

$$e_{p_1} = - e_{p_2} = \cdots = - e_{p_n} \tag{105}$$

Call the value of the individual terms of equations (105) e_p. Then, from equations (95) and (103),

$$e_p = - \frac{a}{n} e_L = - \frac{ue_gR_L}{r_p(\frac{1}{a}2) + \frac{R}{n}L} \tag{106}$$

The plate voltage of the even-numbered tubes is

$$e_b = E_{bb} + e_p = E_{bb} - \frac{ue_gR_L}{r_p(\frac{1}{a}2) + \frac{R}{n}L} \tag{107}$$

The plate voltage of the odd-numbered tubes is

$$e_b = E_{bb} - e_p = E_{bb} + \frac{ue_gR_L}{r_p(\frac{1}{a}2) + \frac{R}{n}L} \tag{108}$$

From equations (96) and (102), the plate current of the even-numbered tubes is

$$i_p = \frac{i_L}{a} = \frac{ue_g}{r_p + \frac{a^2 R_L}{n}} \tag{109}$$

The plate current of the odd-numbered tubes is the negative of equation (109). From equation (109), the value of load resistance as seen by each tube is

$$R_b = \frac{a^2}{n} R_L \tag{110}$$

<u>Non-Linear Analysis</u>.--An analysis of the circuit of Fig. 8 will now be made, assuming large signal operation. With the vacuum tubes operating in the non-linear regions of their static characteristic, the equivalent circuit of Fig. 9 is no longer valid. A graphical analysis will be made using the static characteristics of the tubes.

The a-c operation of the transformer is not affected by the region of operation of the tubes; therefore the transformer equations will be unchanged. These equations are equations (95) and (96).

The following set of equations is obtained from Fig. 8.

$$e_{b_1} = E_{bb} + e_{p_1} \tag{111}$$
$$e_{b_2} = E_{bb} + e_{p_2}$$
$$\cdots\cdots\cdots$$
$$e_{b_n} = E_{bb} + e_{p_n}$$

Or

$$e_{p_1} = e_{b_1} - E_{bb} \tag{112}$$
$$e_{p_2} = e_{b_2} - E_{bb}$$
$$\cdots\cdots\cdots$$
$$e_{p_n} = e_{b_n} - E_{bb}$$

Substituting equations (112) into (95),

$$e_L = \frac{1}{a}(e_{b_2} - e_{b_1} + \ldots + e_{b_n} - e_{b_{n-1}}) \tag{113}$$

Since the tubes are identical, their static characteristics are identical. An inspection of Fig. 8 will show that the quiescent operating points are the same. Thus the quiescent plate currents of the tubes will be equal. Or

$$I_{bo_1} = I_{bo_2} = \ldots = I_{bo_n} \tag{114}$$

Call the value of the individual terms of equations (114) I_{bo}. Then, from Fig. 8,

$$\begin{aligned} i_{b_1} &= I_{bo} + i_{p_1} \\ i_{b_2} &= I_{bo} + i_{p_2} \\ &\cdots\cdots\cdots \\ i_{b_n} &= I_{bo} + i_{p_n} \end{aligned} \tag{115}$$

From equations (96),

$$i_{p_1} = i_{p_3} = \ldots = i_{p_{n-1}} \tag{116}$$

and

$$i_{p_2} = i_{p_4} = \ldots = i_{p_n} \tag{117}$$

Call the value of the individual terms of equations (116) i_p and of equations (117) i_p'. Note from equation (96) that $i_p = - i_p'$. From equations (115) and (116),

$$i_{b_1} = i_{b_3} = \ldots = i_{b_{n-1}} \tag{118}$$

Call the value of the individual terms of equations (118) i_b. From equations (115) and (117),

$$i_{b_2} = i_{b_4} = \ldots = i_{b_n} \tag{119}$$

Call the value of the individual terms of equations (119) i_b'.

Now

$$e_{c_1} = E_{cc} - e_g \tag{120}$$
$$e_{c_2} = E_{cc} + e_g$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$e_{c_n} = E_{cc} + e_g$$

Or

$$e_{c_1} = e_{c_3} = \cdots = e_{c_{n-1}} \tag{121}$$

$$e_{c_2} = e_{c_4} = \cdots = e_{c_n} \tag{122}$$

Thus, from equations (121) and (122), at any instant of time the grid voltages of the even numbered tubes are equal; and at any instant of time the grid voltages of the odd numbered tubes are equal. From equations (118) and (119), at any instant of time the plate currents of the even numbered tubes are equal; and at any instant of time the plate currents of the odd numbered tubes are equal. Therefore, at any instant of time, the plate voltages of the even numbered tubes are equal, and the plate voltages of the odd numbered tubes are equal. Or

$$e_{b_1} = e_{b_3} = \cdots = e_{b_{n-1}} \tag{123}$$

$$e_{b_2} = e_{b_4} = \cdots = e_{b_n} \tag{124}$$

Call the value of the individual terms of equations

(123) e_b and of equations (124) e_b'. Substituting equations (123) and (124) into (113),

$$e_L = \frac{1}{a}\left(\frac{n}{2}\, e_b' - \frac{n}{2}\, e_b\right) = \frac{n}{2a}(e_b' - e_b) \qquad (125)$$

Equation (125) states that the load voltage, and thus the load current, is determined by the difference of the plate voltages of the even-numbered tubes and the odd-numbered tubes. At any instant of time, the values of e_c and i_b will determine the value of e_b on the static characteristic. And at any instant of time, the values of e_c' and i_b' will determine the value of e_b' on the static characteristic. At any instant of time, $e_c + e_c' = 2E_{cc}$. And at any instant of time, i_b will vary from I_{bo} by an amount equal to i_p, and i_b' will vary from I_{bo} by an equal amount in the opposite direction.

The composite characteristic is formed in the following manner: The values of E_{bb} and E_{cc} are chosen. Since the coils of the transformer are assumed to have zero resistance, then the value of I_{bo} can be obtained directly from the static characteristic of the tubes. The value of I_{bo} will be the value of i_b at the point $e_c = E_{cc}$ and $e_b = E_{bb}$. A value for both e_c and i_b is chosen, and the corresponding value of e_b is obtained from the static characteristic. For $e_c = E_{cc} - e_g$, the corresponding value of e_c' is $e_c' = E_{cc} + e_g$. And for

$i_b = I_{bo} + i_p$, the corresponding value of i_b' is $i_b' = I_{bo} - i_p$. Therefore the value of e_b' corresponding to each e_b can be obtained from the static characteristic. By equation (125) the load voltage varies as $(e_b' - e_b)$. A sufficient number of points can be obtained by the method described above to plot $(e_b' - e_b)$ vs i_b for values of constant e_c. The resultant plot will be the composite characteristic for the circuit of Fig. 8.

From equations (101) and (125),

$$e_L = i_L R_L = \frac{n}{2a}(e_b' - e_b) \tag{126}$$

Solving for i_L,

$$i_L = \frac{n}{2aR_L}(e_b' - e_b) \tag{127}$$

Substituting equations (96) into (127) and solving for i_p,

$$i_p = \frac{n}{2a^2R_L}(e_b' - e_b) \tag{128}$$

The composite characteristic is plotted in terms of i_b. And $i_b = I_{bo} + i_p$. Thus, since I_{bo} is a constant, i_b will vary directly as i_p varies. Thus equation (128) may be plotted on the composite characteristic and this plot will be the path of operation. It will be noted that this plot is a straight line. If the voltage scale

$(e_b' - e_b)$ is multiplied by the factor n/2, then the load line will have a slope

$$\text{slope} = -\frac{1}{a^2 R_L} \tag{129}$$

on the composite characteristic.

When $i_p = 0$, then $i_b = I_{bo}$ and $(e_b' - e_b) = 0$. This is a point on the load line. This point is known; and since the slope of the load line is known, then the load line can be constructed on the composite characteristic.

A graphical method for the construction of the composite characteristic will now be devised. Consider Fig. 10. In Fig. 10, a static characteristic of the vacuum tubes has been turned upside down and placed adjacent to another static characteristic of the vacuum tubes. The points I_{bo} on the i_b scale on both characteristics have been made to coincide. With the characteristics in this position, the composite characteristic can be constructed. Consider the line abcd. Point d on the right hand characteristic is the value of e_b for a certain $i_b = I_{bo} + i_p$ and $e_c = E_{cc} - e_g$. Point a on the left hand characteristic is the value of e_b' for $i_b' = I_{bo} - i_p$ and $e_c' = E_{cc} + e_g$. Thus $(e_b' - e_b)$ will be the point on the composite characteristic for this certain i_b and e_c. And $e_b' = \overline{ca}$ and $e_b = \overline{cd}$. Thus

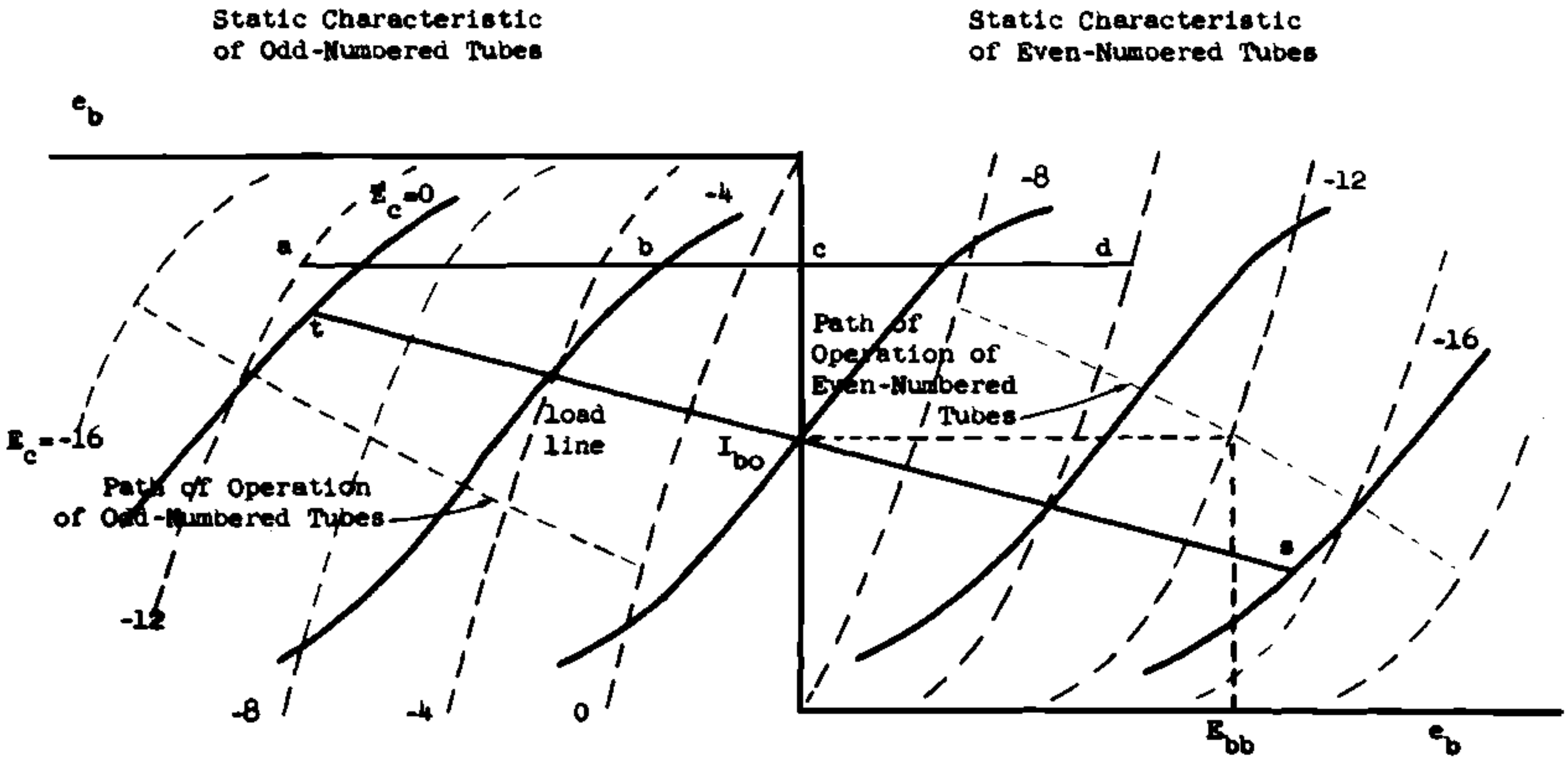


Fig. 10.--Graphical Method for Construction of Composite Characteristic of Push-Pull Parallel Magnetic Circuit.

$$(e_b' - e_b) = \overline{ca} - \overline{cd} = \overline{cb} \tag{130}$$

Thus point b is a point of the composite characteristic. This method can be used to locate a sufficient number of points to plot the composite characteristic. Once the composite characteristic has been plotted, then the $(e_b' - e_b)$ scale must be multiplied by a factor of n/2.

Equation (128) can also be written

$$(i_b - I_{bo}) = \frac{(e_b' - e_b)}{(\frac{2}{n})a^2R_L} \tag{131}$$

The factor n/2 may be considered as part of the load resistance. Then the composite characteristic is formed as explained above, but the $(e_b' - e_b)$ scale is not multiplied by the factor n/2. Instead the slope of the load line is changed to

$$\text{slope} = -\frac{\frac{n}{2}}{a^2R_L} \tag{132}$$

This composite characteristic is the equivalent of the first composite characteristic. In the first composite characteristic the voltage scale is expanded to allow for the factor n/2. In the second composite characteristic the slope of the load line is expanded to allow for the factor n/2. In either case the path of operation on the composite characteristic is the same.

The effect of increasing n will now be considered. Consider the second composite characteristic explained above. From equation (132), if n is increased, then the slope of the load line is increased. Thus the value of $(e_b' - e_b)$ will decrease. Consider the case where as the number of tubes is increased by the factor n, the load resistance is made equal to nR_L. Then, from equation (132), the slope of the load line will not change, and $(e_b' - e_b)$ will not change. Also, from the linear analysis, equation (110), the value of load resistance as seen by each tube will remain constant if the actual load resistance is made equal to nR_L. The power output of the circuit is

$$P_L = i_L^2 R_L \tag{133}$$

From equations (127) and (133),

$$P_L = \frac{n^2}{4a^2 R_L}(e_b' - e_b)^2 = \frac{n}{4a^2 R_1}(e_b' - e_b)^2 \tag{134}$$

where

$$R_1 = nR_L \tag{135}$$

Thus, from equation (134), as the number of tubes is increased, the power output is increased by the same factor, provided that the load resistance is given by equation (135).

If R_L is held constant as the number of tubes is increased, then $(e_b' - e_b)$ will decrease. From equation (134), the n^2 term will tend to increase the power out, but the $(e_b' - e_b)^2$ term will tend to decrease the power out. The actual effect of increasing the number of tubes for constant R_L will therefore depend on the characteristics of the tubes used, the value of R_L, and the number n. However, it will be noted that if the number of tubes is increased, the circuit will be capable of a higher power output, because of the increased number of tubes.

From the linear analysis, equation (104), if the number of tubes is increased, the voltage gain of the circuit will be increased by the same factor provided the load resistance is given by equation (135). However, from equation (104), if the load resistance is held constant as the number of tubes is increased, the voltage gain of the circuit will increase by a factor less than the factor by which the number of tubes is increased. The increase is determined by the values of a, r_p and R_L.

The treatment of the circuit of Fig. 8 can be made analogous to that of a single-tube circuit incorporating a "composite tube". The operation of the composite tube circuit will be described by equations (121), (126), (127) and (128). All the vacuum tubes except tube number one are removed from the circuit of Fig. 8. The grid voltage of the remaining tube is given by equation (121). The plate voltage of the remaining tube, which is the composite tube, is

$$e_{b_c} = E_{bb_c} + e_{p_c} = E_{bb} + \frac{n}{2}(e_b' - e_b) \qquad (136)$$

Thus the static characteristic of the composite tube is the composite characteristic of Fig. 8. The a-c component of plate current is given by equation (128), which is the equation for the a-c component of plate current of the even-numbered tubes. For equations (95) and (96) to be satisfied, the primary coils of the tubes that were removed must be short-circuited. Under these conditions, the composite tube circuit satisfies equations (121), (126), (127) and (128).

If the number of tubes of Fig. 8 is two, then the operation of the circuit is comparable to that of a two-tube push-pull circuit. When one tube is "pushing" with respect to voltage, the other tube is "pulling" with respect to voltage. This is shown graphically in Fig. 11, and is explained in the distortion analysis. These two tubes are effectively in series with the load. From equations (95), (123), (124) and (125) the effect of the transformer is to put n/2 of these two-tube circuits in series with the load. That is, except for the factor of the turns ratio, the output voltage is the sum of the output voltages of n/2 of these two-tube circuits. And the output current is the same as the currents of the even-numbered tubes, and the negative of the currents of the odd-numbered tubes.

Distortion Analysis.--An analysis will now be made to determine the amount of distortion introduced by the circuit. The analysis will be made assuming a sinusoidal input signal.

The path of operation of the tubes can be determined from the composite characteristic of the circuit. On the composite characteristic, the intersection of a constant e_c curve with the load line will give a value of i_p. If these values of i_p vs e_c are plotted on the static characteristic, the result will be the path of operation. The paths of operation of both the even-numbered tubes and the odd-numbered tubes are shown in Fig. 10.

It will be noted that the paths of operation of both the even-numbered tubes and the odd-numbered tubes are identical. In the graphical method for construction of the composite characteristic as shown in Fig. 10, the composite characteristic is symmetrical with respect to the point I_{bo}. That is, suppose a line is drawn through the point I_{bo} in such a manner as to intersect one of the composite characteristic curves. Then this point of intersection will be a point defined by a certain

i_{b_x} and e_{c_x}. Now if the line is traveled in the opposite direction from I_{bo}, the line will intersect a composite characteristic curve at another point which is the same distance from the point I_{bo} as is the first point. And this point will be defined by $i_{b_y} = i'_{b_x}$ and $e_{c_y} = e'_{c_x}$.

Consider the load line, which is drawn through the point I_{bo} in Fig. 10. The point s will give a certain i_{b_s} and e_{c_s}, which is a point of operation of the even-numbered tubes. The point t will give a certain $i'_{b_t} = i_{b_s}$ and $e'_{c_t} = e_{c_s}$, which is a point of operation of the odd-numbered tubes. Thus the points of operation of the even-numbered tubes and the odd-numbered tubes are identical. However, it will be noted that when the even-numbered tubes are at a point on the path of operation defined by $e_c = E_{cc} - e_g$, then the odd-numbered tubes are at a point on an identical path of operation defined by $e'_c = E_{cc} + e_g$. Thus the shapes of the generated voltage waves e_b and e'_b will be identical, but e'_b will be displaced from e_b by 180° at the fundamental frequency.

The voltage wave e_b is a periodic wave that can be represented by a Fourier Series.

$$e_b = A_o + A_1\sin(wt + \theta_1) + A_2\sin(2wt + \theta_2) + A_3\sin(3wt + \theta_3) + \dots \tag{137}$$

Since e'_b has the same wave shape as e_b, then the amplitudes of the different frequencies must be the same. And since e_b' is 180° out of phase with e_b with respect to the fundamental frequency, then the n<u>th</u> harmonic of e_b' is $n180^\circ$ out of phase with the n<u>th</u> harmonic of e_b. Or

$$e_b' = A_o + A_1 \sin(wt + \theta_1 + 180^\circ) + A_2 \sin(2wt + \theta_2 + 360^\circ) + A_3 \sin(3wt + \theta_3 + 540^\circ) + \ldots \tag{138}$$

Now

$$\sin(nwt + \theta + n180^\circ) = -\sin(nwt + \theta) \tag{139}$$

where n is odd. And

$$\sin(nwt + \theta + n180^\circ) = \sin(nwt + \theta) \tag{140}$$

where n is even. Thus

$$e_b' = A_o - A_1 \sin(wt + \theta_1) + A_2 \sin(2wt + \theta_2) - A_3 \sin(3wt + \theta_3) + \ldots \tag{141}$$

And

$$e_b' - e_b = -2A_1 \sin(wt + \theta_1) - 2A_3 \sin(3wt + \theta_3) - \ldots \tag{142}$$

Since e_L is directly proportional to $(e_b' - e_b)$, then the load voltage contains only the odd harmonics. And since i_L is directly proportional to e_L, then the load current contains only the odd harmonics.

From equations (96),

$$i_L = ai_p = - ai_p' \tag{143}$$

Thus only the odd harmonics are contained in the plate currents of the tube. The a-c plate current of the even-numbered tubes is 180° out of phase with the a-c plate current of the odd-numbered tubes. The plate currents add in the power supply; and since these currents contain only odd harmonics, these harmonics will cancel and the power supply current will have no harmonic content.

The cancellation of harmonics is shown graphically in Fig. 11. The left hand set of static characteristics represents the odd-numbered tubes, and the right hand set of characteristics represents the even-numbered tubes. Since the output voltage is proportional to $(e_b' - e_b)$, the right hand set of characteristics has been reversed. Thus the two waves may be added directly. From this graphical representation it is seen that the fundamental components of the voltages add, and the second harmonic components subtract, giving a resultant output voltage that is free of second harmonic content.

Variations in E_{bb}, such as ripple voltage in the power supply and variations in the power-supply voltage caused by poor regulation, are applied simultaneously to all tubes. It is seen from equations (111) that these variations cause equal increments in the plate voltages

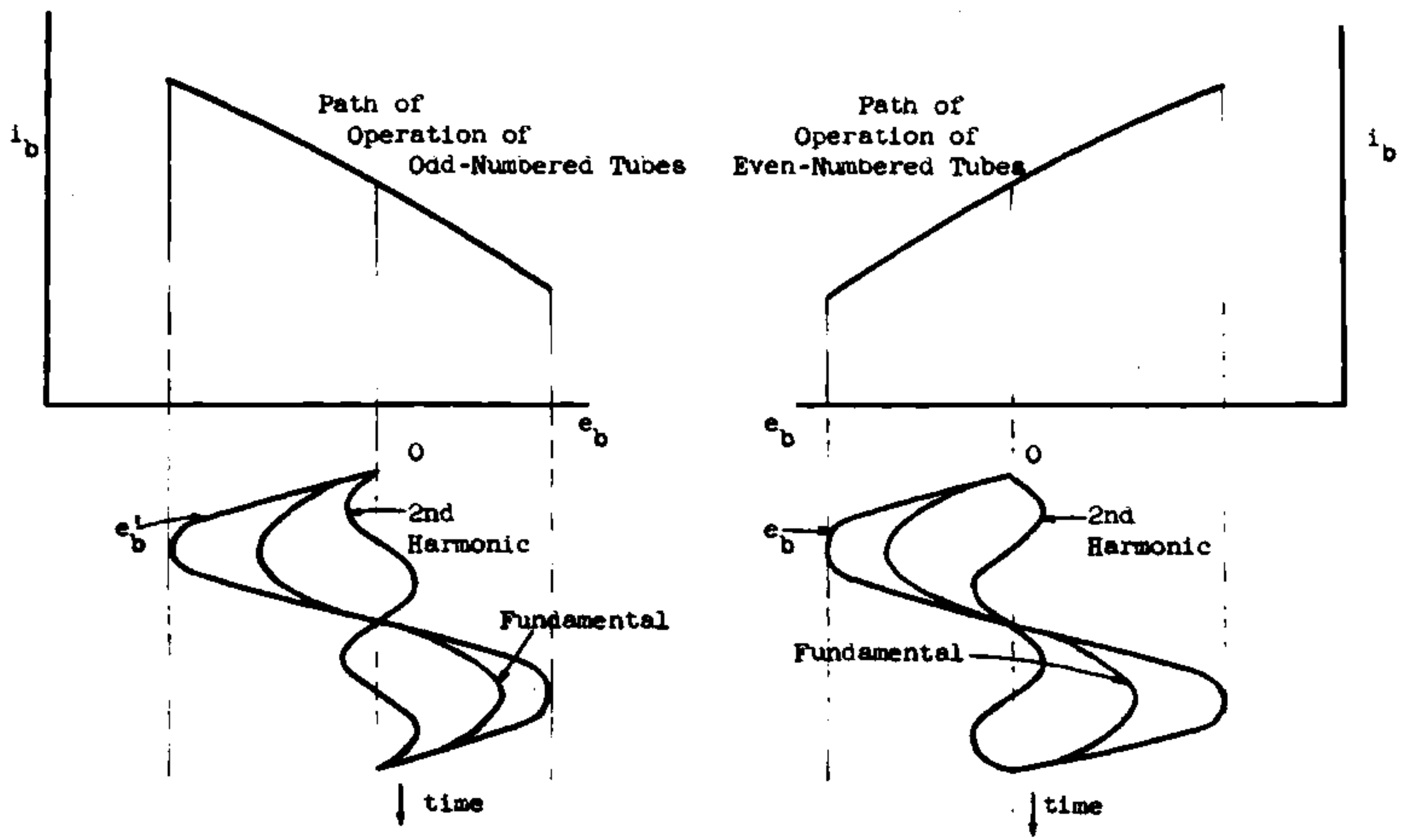


Fig. 11.--Graphical Representation of Non-Linear Operation of Push-Pull Parallel Magnetic Circuit.

of both the even-numbered tubes and the odd-numbered tubes. From equation (125), the load voltage is proportional to the difference between the plate voltages of the even-numbered tubes and the odd-numbered tubes. Thus the effect of the variations in E_{bb} will be cancelled in the output.

Magnetization of the Core.--An analysis will now be made to determine the extent of d-c magnetization of the core of the output transformer. The magnetic circuit of the output transformer is shown in Fig. 12. It is assumed that the legs of the transformer core are of the same uniform cross-sectional area, and that the lengths of the magnetic paths from a common point a at the top node to a common point b at the bottom node are equal. Then, for each closed magnetic path,

$$F = \sum Ni = \oint \overline{H} \cdot d\overline{s} \qquad (144)$$

where F is the total magnetomotive force acting around the closed path, $\sum Ni$ is the sum of the magnetomotive forces of the coils on the closed path, and $\overline{H}$ is the magnetic intensity along the closed path. In equation (144) the directions of the different magnetomotive forces of the coils must be taken into consideration in the summation.

Two simplifying assumptions will be made. The first assumption is that the direction of the magnetic

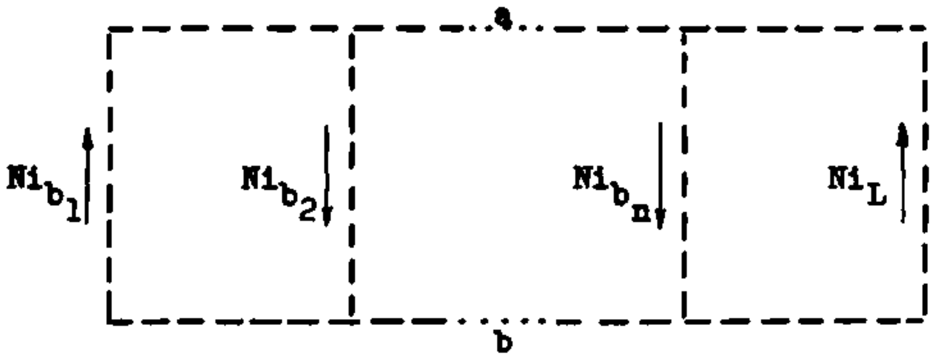


Fig. 12. Magnetic Circuit of the Transformer of the Push-Pull Parallel Magnetic Circuit.

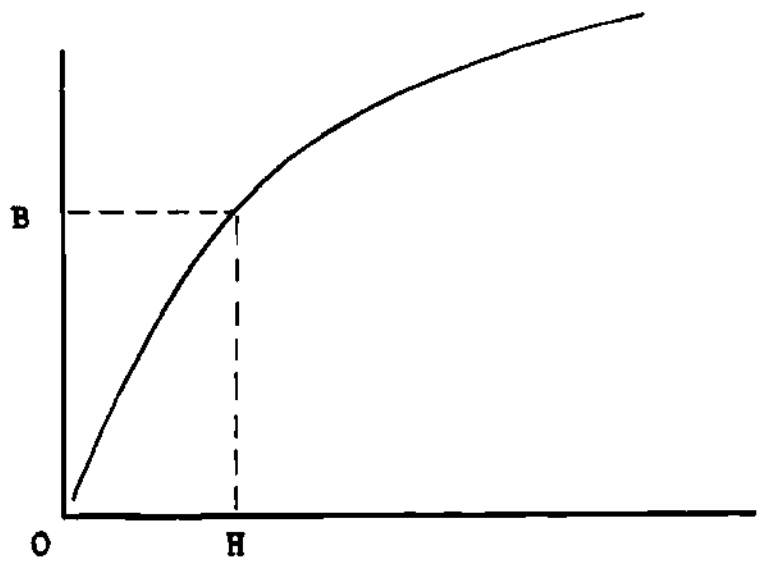


Fig. 13. Sample Magnetization Curve.

intensity is parallel to the direction of the magnetic path. Or

$$\oint \bar{H} \cdot d\bar{s} = \oint H ds \qquad (145)$$

The second assumption is that the magnetic intensity along any path ℓ between a and b is independent of the distance along ℓ . Consider the first coil leg and the load coil leg in Fig. 12 as the closed magnetic path. The second assumption says that, for this path,

$$\oint H d\ell = \int_b^a H_1 d\ell + \int_a^b H_L d\ell = \qquad (146)$$

$$H_1 \int_b^a d\ell - H_L \int_b^a d\ell = (H_1 - H_L)\ell$$

Or, from equation (144),

$$N i_{b_1} - N_L i_L = (H_1 - H_L)\ell \qquad (147)$$

It will be noted that the direction of the magnetomotive force in the even-numbered coils is opposite to that of the odd-numbered coils. Or, for the second coil leg and the load coil leg in Fig. 12,

$$- N i_{b_2} - N_L i_L = \int_b^a (-H_2) d\ell + \int_a^b H_L d\ell = \qquad (148)$$

$$(- H_2 - H_L)\ell$$

The magnetic intensity H_2 has a negative sign due to its direction being assumed opposite to that of H_1.

This direction has been assumed opposite to that of H_1 because the direction of Ni_{b_2} is opposite to that of Ni_{b_1}.

From equations (115), (116) and (118),

$$i_{b_1} = i_{b_3} = \dots = i_{b_{n-1}} = I_{bo} + i_p \qquad (149)$$

From equation (96) and (116),

$$i_L = \frac{N}{N_L} i_p \qquad (150)$$

Substituting equations (149) and (150) into (147),

$$N(I_{bo} + i_p) - N_L(\frac{N}{N_L} i_p) = (H_1 - H_L)\ell \qquad (151)$$

Or

$$NI_{bo} = (H_1 - H_L)\ell \qquad (152)$$

From equations (115), (117) and (119),

$$i_{b_2} = i_{b_4} = \dots = i_{b_n} = I_{bo} - i_p \qquad (153)$$

Substituting equations (150) and (153) into (148),

$$- N(I_{bo} - i_p) - N_L(\frac{N}{N_L} i_p) = - (H_2 + H_L)\ell \qquad (154)$$

Or

$$NI_{bo} = (H_2 + H_L)\ell \qquad (155)$$

By the same manner used in obtaining equations (152) and (155),

$$NI_{bo} = (H_3 - H_L)\ell \qquad (156)$$

$$NI_{bo} = (H_4 + H_L)\ell$$

$$\cdots\cdots\cdots\cdots$$

$$NI_{bo} = (H_{n-1} - H_L)\ell$$

$$NI_{bo} = (H_n + H_L)\ell$$

Thus, from equations (152), (155), and (156),

$$H_1 = H_3 = \ldots = H_{n-1} \qquad (157)$$

$$H_2 = H_4 = \ldots = H_n \qquad (158)$$

Call the value of the individual terms of equations (157) H and of (158) H'. Then equations (152) and (155) become

$$NI_{bo} = (H - H_L)\ell \qquad (159)$$

$$NI_{bo} = (H' + H_L)\ell \qquad (160)$$

Assuming no leakage flux, the sum of the flux into the top node is

$$\phi_1 - \phi_2 + \ldots + \phi_{n-1} - \phi_n + \phi_L = 0 \qquad (161)$$

Since each leg of the transformer is constructed of the same material, then each leg has the same magnetization curve. Or, for each leg, the relationship between the flux density B and the magnetic intensity H is the same. Thus, considering equations (157),

$$B_1 = B_3 = \dots = B_{n-1} \tag{162}$$

Considering equations (158),

$$B_2 = B_4 = \dots = B_n \tag{163}$$

Call the value of the individual terms of equations (162) B and of equations (163) B'.

For any magnetic circuit,

$$\phi = \int_S \bar{B} \cdot d\bar{a} \tag{164}$$

For the magnetic circuit of Fig. 12, the flux density B is assumed to be normal to the cross-sectional area of the legs, and to be uniformly distributed over the cross-sectional area. Then equation (164) becomes

$$\phi = \int_S \bar{B} \cdot d\bar{a} = BA \tag{165}$$

where A is the cross-sectional area of the legs. Thus, from equations (162), (163) and (165),

$$\phi_1 = \phi_3 = \dots = \phi_{n-1} \tag{166}$$

$$\phi_2 = \phi_4 = \dots = \phi_n \tag{167}$$

Call the value of the individual terms of equations (166) ϕ and of equations (167) ϕ'. Substituting equations (166) and (167) into (161),

$$\phi_L = \frac{n}{2}(\phi' - \phi) \tag{168}$$

Or

$$B_L = \frac{n}{2}(B' - B) \tag{169}$$

Equations (159), (160) and (169) describe the magnetic conditions of the core of Fig. 12. The actual magnetization of the core is found by using these equations in conjunction with the magnetization curve of the core material. A sample magnetization curve is shown in Fig. 13.

Solving equations (159) and (160) for H_L,

$$2H_L = H - H' \tag{170}$$

Now H is either greater than, equal to, or less than H'. Suppose that H is greater than H'. Then, from equation (170), H_L is positive. From Fig. 13 it is seen that if H is greater than H', then B is greater than B'. Thus, from equation (169), B_L is negative. But B and H are related by

$$B = \mu H \tag{171}$$

and B and H must have the same sign. Thus H cannot be greater than H'.

Suppose H is less than H'. Then, from equation (170), H_L is negative. Now B is less than B'. Thus, from equation (169), B_L is positive. But H_L and B_L must have the same sign. Thus H cannot be less than H'.

Then the only possibility left is that H and H' are equal. And from equation (170), H_L is zero. Thus equations (159) and (160) become

$$NI_{bo} = H\ell = H'\ell \qquad (172)$$

Thus the d-c magnetization of the load coil leg is zero, and the d-c magnetization of the other legs is given by equation (172). Since NI_{bo} and ℓ are known, then H is calculated from equation (172) and B is found from the magnetization curves.

CHAPTER VI

COMMON-INPUT PARALLEL MAGNETIC CIRCUIT

Linear Analysis.--The fourth circuit to be investigated is the circuit of Fig. 14. It will be noted that the primary coils of the output transformer are magnetically in parallel, and that the polarity of all primary coils is the same. Assume the turns of the primary coils to be equal, and let a be the ratio of turns of one primary coil to the turns of the secondary coils. Then the equations of the transformer are, from equations (12) and (14),

$$e_L = -\frac{1}{a}(e_{p_1} + e_{p_2} + \dots + e_{p_n}) \qquad (173)$$

$$i_L = ai_{p_1} = ai_{p_2} = \dots = ai_{p_n} \qquad (174)$$

If the operation of the vacuum tubes is considered to be in the linear regions of their static characteristic, then Fig. 15 is the a-c equivalent circuit of Fig. 14. It will be noted that the grid-signal voltages of the vacuum tubes are equal and in phase. For this circuit, n+1 loop equations can be written. Consider the n equations obtained by writing loop equations for each tube circuit.

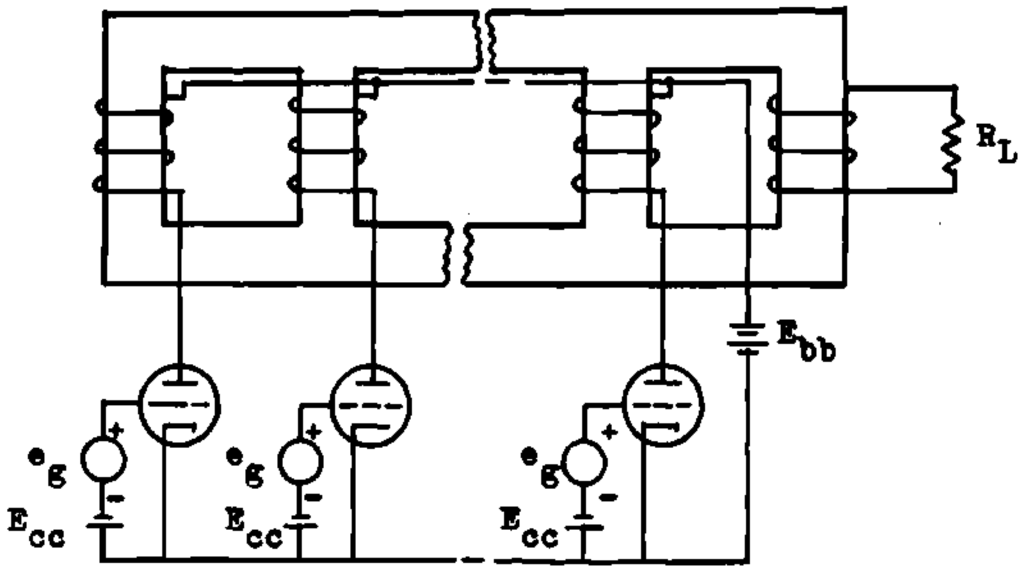


Fig. 14. Common-Input Parallel Magnetic Circuit.

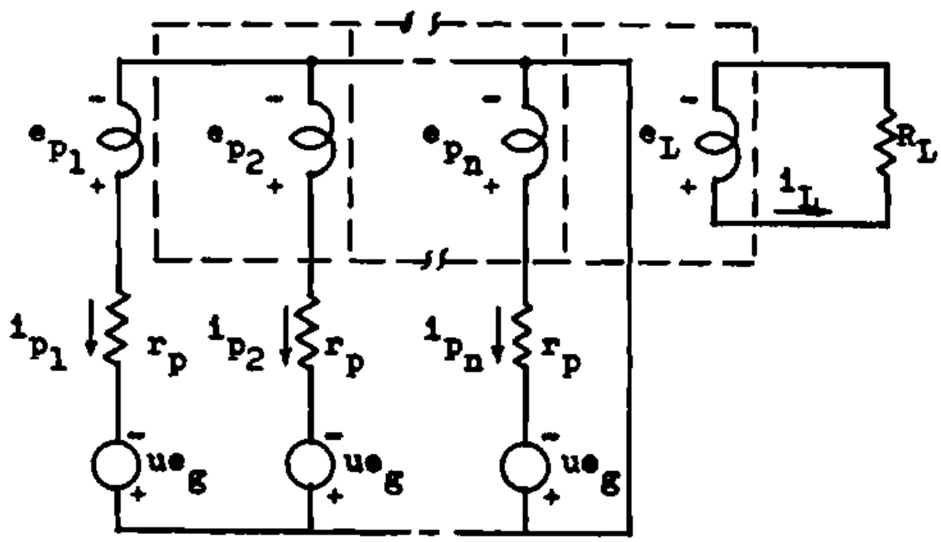


Fig. 15. Equivalent A-C Circuit of Common-Input Parallel Magnetic Circuit.

$$ue_g - i_{p_1}r_p + e_{p_1} = 0 \qquad (175)$$
$$ue_g - i_{p_2}r_p + e_{p_2} = 0$$
$$\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot$$
$$ue_g - i_{p_n}r_p + e_{p_n} = 0$$

Adding these equations,

$$nue_g - r_p(i_{p_1} + i_{p_2} + \ldots + i_{p_n}) + \qquad (176)$$
$$(e_{p_1} + e_{p_2} + \ldots + e_{p_n}) = 0$$

From equations (174),

$$i_{p_1} = i_{p_2} = \ldots = i_{p_n} \qquad (177)$$

Call the value of the individual terms of equations (177) i_p. Substituting equations (173) and (177) into (176),

$$nue_g - ni_pr_p - ae_L = 0 \qquad (178)$$

And

$$e_L = i_LR_L \qquad (179)$$

Substituting equations (174) and (179) into (178)

$$nue_g - \frac{n}{a}\, i_Lr_p - ai_LR_L = 0 \qquad (180)$$

Solving for i_L,

$$i_L = \frac{ue_g}{(\frac{1}{a})r_p + \frac{a}{n} R_L} \qquad (181)$$

and

$$e_L = \frac{ue_g R_L}{(\frac{1}{a})r_p + \frac{a}{n} R_L} \qquad (182)$$

The gain A is

$$A = \frac{e_L}{e_g} = \frac{uR_L}{(\frac{1}{a})r_p + \frac{a}{n} R_L} \qquad (183)$$

From equations (175) and (177),

$$e_{p_1} = e_{p_2} = \dots = e_{p_n} \qquad (184)$$

Call the value of the individual terms of equations (184) e_p. Then equation (173) becomes

$$e_L = -\frac{1}{a}(ne_p) = -\frac{n}{a} e_p \qquad (185)$$

Solving for e_p and using equation (182),

$$e_p = -\frac{a}{n} e_L = -\frac{aue_g R_L}{\frac{n}{a} r_p + aR_L} \qquad (186)$$

The plate voltage of the tubes is

$$e_b = E_{bb} + e_p \qquad (187)$$

Substituting equation (186) into (187),

$$e_b = E_{bb} - \frac{aue_g R_L}{\frac{n}{a} r_p + aR_L} \tag{188}$$

The a-c plate current of the tubes is, from equations (174) and (181)

$$i_p = \frac{i_L}{a} = \frac{ue_g}{r_p + (\frac{1}{n})a^2 R_L} \tag{189}$$

Thus the value of load resistance as seen by each tube is

$$R_b = \frac{a^2}{n} R_L \tag{190}$$

Non-Linear Analysis.--An analysis of the circuit of Fig. 14 will now be made, assuming large signal operation. With the vacuum tubes operating in the non-linear regions of their static characteristic, the equivalent circuit of Fig. 15 is no longer valid. A graphical analysis will be made using the static characteristics of the tubes.

The a-c operation of the transformer is not affected by the region of operation of the tubes; therefore the transformer equations will be unchanged. These equations are equations (173) and (174).

The following equations are obtained from Fig. 14.

$$\begin{aligned} e_{b_1} &= E_{bb} + e_{p_1} \\ e_{b_2} &= E_{bb} + e_{p_2} \\ &\cdots\cdots\cdots \\ e_{b_n} &= E_{bb} + e_{p_n} \end{aligned} \tag{191}$$

Or

$$\begin{aligned} e_{p_1} &= e_{b_1} - E_{bb} \\ e_{p_2} &= e_{b_2} - E_{bb} \\ &\cdots\cdots\cdots \\ e_{p_n} &= e_{b_n} - E_{bb} \end{aligned} \tag{192}$$

Substituting equations (192) into (173),

$$e_L = -\frac{1}{a}(e_{b_1} + e_{b_2} + \dots + e_{b_n} - nE_{bb}) \tag{193}$$

Since the tubes are identical, their static characteristics are identical. An inspection of Fig. 14 will show that the quiescent operating points of the tubes are the same. Thus the quiescent plate currents of the tubes will be equal. Or

$$I_{bo_1} = I_{bo_2} = \dots = I_{bo_n} \tag{194}$$

Call the value of the individual terms of equations (194) I_{bo}. Then, from Fig. 14,

$$
\begin{aligned}
i_{b_1} &= I_{bo} + i_{p_1} \\
i_{b_2} &= I_{bo} + i_{p_2} \\
&\cdots\cdots \\
i_{b_n} &= I_{bo} + i_{p_n}
\end{aligned}
\tag{195}
$$

From equations (174),

$$i_{p_1} = i_{p_2} = \ldots = i_{p_n} \tag{196}$$

Thus, from equations (195) and (196),

$$i_{b_1} = i_{b_2} = \ldots = i_{b_n} \tag{197}$$

Call the value of the individual terms of equation (197) i_b. Also

$$
\begin{aligned}
e_{c_1} &= E_{cc} - e_g \\
e_{c_2} &= E_{cc} - e_g \\
&\cdots\cdots \\
e_{c_n} &= E_{cc} - e_g
\end{aligned}
\tag{198}
$$

Thus

$$e_{c_1} = e_{c_2} = \ldots = e_{c_n} \tag{199}$$

From equation (199), at any instant of time the grid voltages of the tubes are equal. And from equations (197), at any instant of time the plate currents of the tubes are equal. Thus, at any instant of time, the plate voltages of the tubes are equal. Or

$$e_{b_1} = e_{b_2} = \cdots = e_{b_n} \qquad (200)$$

Call the value of the individual terms of equations (200) e_b. Then, from equation (193),

$$e_L = -\frac{1}{a}(ne_b - nE_{bb}) = -\frac{n}{a}(e_b - E_{bb}) \qquad (201)$$

From equations (174), (195) and (197),

$$i_L = ai_p = a(i_b - I_{bo}) \qquad (202)$$

And from equations (179), (201) and (202),

$$e_L = i_L R_L = aR_L(i_b - I_{bo}) = -\frac{n}{a}(e_b - E_{bb}) \qquad (203)$$

Or

$$(i_b - I_{bo}) = -\frac{n(e_b - E_{bb})}{a^2 R_L} \qquad (204)$$

From equations (198) and (199),

$$e_{c_1} = e_{c_2} = \cdots = e_{c_n} = E_{cc} - e_g \qquad (205)$$

Equations (204) and (205) describe the operation of the circuit of Fig. 14. Equation (204) is the relationship between tube currents and tube voltages to be used to construct a composite characteristic for the circuit. However, it will be noted that equation (204) is the relationship for the operation of one of the tubes of the circuit in which the voltage has been multiplied by a factor of n. Thus the composite characteristic for the circuit will be the static characteristic of the tubes with the voltage scale multiplied by a factor of n.

It will be noted from equation (204) that the path of operation is a straight line with a slope of

$$\text{slope} = -\frac{1}{a^2 R_L} \tag{206}$$

on the composite characteristic. When no signal is applied to the circuit, $i_b = I_{bo}$. Since the transformer is assumed ideal, there is no resistance associated with the windings. Thus $e_b = E_{bb}$. Since the value of E_{cc} is known, then the value of I_{bo} can be found from the composite characteristic of the circuit. The value of I_{bo} will be the value of i_b at $ne_b = nE_{bb}$ and $e_c = E_{cc}$. This point is a point of the path of operation. Thus, with one point and the slope of the path of operation known, the load line can be constructed on the composite characteristic.

The composite characteristic for a circuit of 6J5 vacuum tubes is shown in Fig. 16. The path of operation is also shown for a value of R_L.

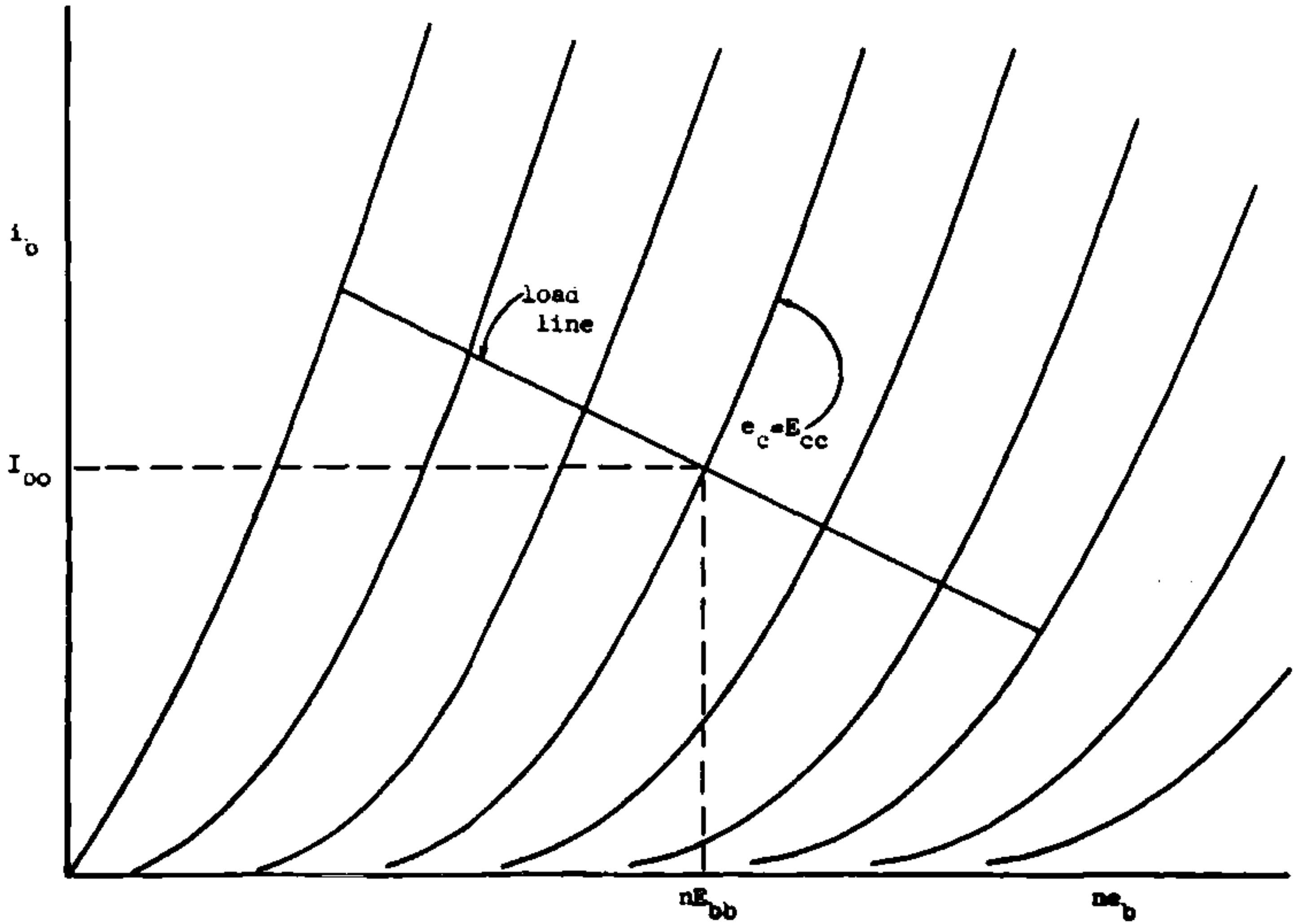


Fig. 16.--Composite Characteristic for Common-Input Parallel Magnetic Circuit.

Equation (204) can also be written

$$(i_b - I_{bo}) = - \frac{(e_b - E_{bb})}{(\frac{1}{n})a^2 R_L} \qquad (207)$$

Thus the composite characteristic can also be represented by the static characteristic of the tubes with the original voltage and current scales, but the slope of the load line will then be

$$\text{slope} = - \frac{n}{a^2 R_L} \qquad (208)$$

This composite characteristic is the equivalent of the first composite characteristic. In the first composite characteristic, the voltage scale is expanded to allow for the factor n. In the second composite characteristic the slope of the load line is expanded to allow for the factor n. In either case the path of operation on the composite characteristic is the same.

The treatment of the circuit of Fig. 14 can be made analogous to that of a single-tube circuit incorporating a "composite tube". In Fig. 14, if all the tubes except one are removed from the circuit, if all the primary coils are short-circuited except the one of the remaining tube, and if the plate voltage of the remaining tube and the voltage of the plate power supply are multiplied by a factor of n, then the remaining tube will be the composite tube. This circuit will satisfy the equations of the original circuit. These equations are equations (204) and (205).

The effect of increasing n will now be considered. Consider the second composite characteristic explained. From equation (208), if n is increased, then the slope of the load line is increased. Thus the value of $(e_b - E_{bb})$ will decrease. Consider the case where as the number of tubes is increased by the factor n, the load resistance is made equal to nR_L. Then, from equation (208), the slope of the load line will not change, and $(e_b - E_{bb})$ will not change. Also, from the linear analysis, equation (190), the value of load resistance as seen by each tube will remain constant if the actual load resistance is made equal to $2R_L/n$. The power output of the circuit is

$$P_L = i_L^2 R_L \qquad (209)$$

From equations (202), (204) and (209),

$$P_L = \frac{n^2(e_b - E_{bb})^2}{R_L} = \frac{n(e_b - E_{bb})^2}{R_1} \qquad (210)$$

where

$$R_1 = nR_L \qquad (211)$$

Thus, from equation (210), as the number of tubes is increased, the power output is increased by the same factor, provided that the load resistance is given by equation (211).

If R_L is held constant as the number of tubes is increased, then $(e_b - E_{bb})$ will decrease. From equation (210), the n^2 term will tend to increase the power out, but the $(e_b - E_{bb})^2$ term will tend to decrease the power out. The actual effect of increasing the number of tubes for constant R_L will therefore depend on the characteristics of the tubes used, the value of R_L, and the number n. However, it will be noted that if the number of tubes is increased, the circuit will be capable of a higher power output, because of the increased number of tubes.

From the linear analysis, equation (183), if the number of tubes is increased, the voltage gain of the circuit will be increased by the same factor provided the load resistance is given by equation (211). However, from equation (183), if the load resistance is held constant as the number of tubes is increased, the voltage gain of the circuit will increase by a factor less than the factor by which the number of tubes is increased. This increase is determined by the values of a, r_p and R_L.

It will be noted from equations (201) and (202) that the effect of the transformer is to put the vacuum tubes in series with the load. That is, except for the factor of the turns ratio, the load current is equal to the current of each tube, and the load voltage is the sum of the output voltages of all the tubes.

There will be no cancellation of harmonics in this circuit. Thus the distortion in the output will be comparable to that of a single tube amplifier. Also there will be no cancellation of harmonics in the power supply, since the currents add in the power supply.

Magnetization of the Core.--An analysis will now be made to determine the extent of d-c magnetization of the core of the output transformer. The magnetic circuit of the output transformer is shown in Fig. 17. It is assumed that the legs of the transformer core are of the same uniform cross-sectional area, and that the lengths of the magnetic paths from a common point a at the top node to a common point b at the bottom node are equal. Then, for each closed magnetic path,

$$F = \sum Ni = \oint \bar{H} \cdot d\bar{s} \tag{212}$$

where F is the total magnetomotive force acting around the closed path, $\sum Ni$ is the sum of magnetomotive forces of the coils on the closed path, and $\bar{H}$ is the magnetic intensity along the closed path. In equation (212) the directions of the different magnetomotive forces of the coils must be taken into consideration in the summation.

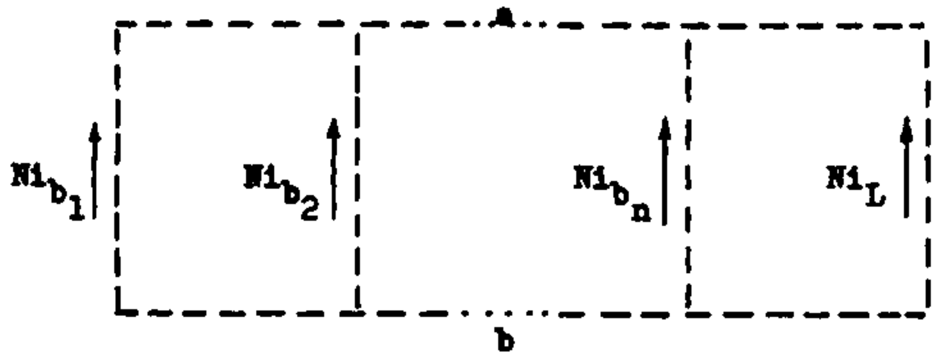


Fig. 17. Magnetic Circuit of the Transformer of the Common-Input Parallel Magnetic Circuit.

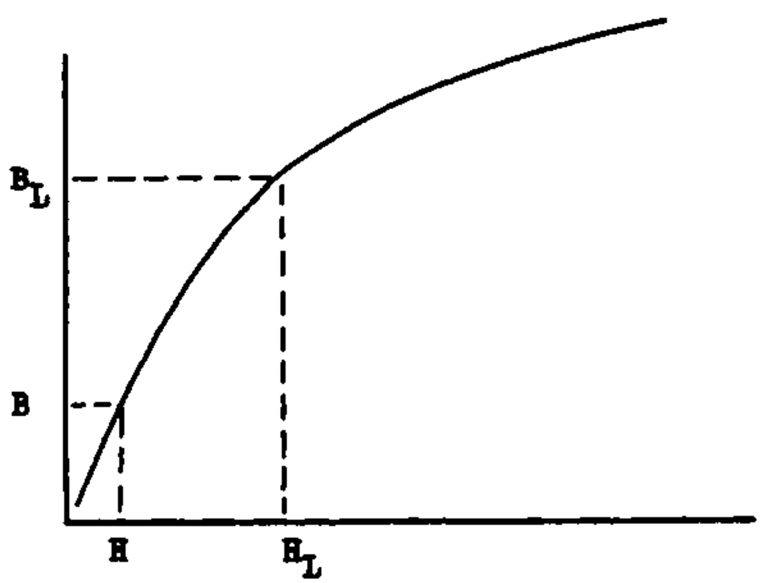


Fig. 18. Sample Magnetization Curve.

Two simplifing assumptions will be made. The first assumption is that the direction of the magnetic intensity is parallel to the direction of the magnetic path. Or

$$\oint \bar{H}\cdot d\bar{s} = \oint H ds \tag{213}$$

The second assumption is that the magnetic intensity along any path ℓ between a and b is independent of the distance along ℓ. Consider the first coil leg and the load coil leg in Fig. 17 as the closed magnetic path. The second assumption says that, for this path,

$$\oint H d\ell = \int_b^a H_1 d\ell + \int_a^b H_L d\ell = \tag{214}$$

$$H_1 \int_b^a d\ell - H_L \int_b^a d\ell = (H_1 - H_L)\ell$$

Or, from equation (212),

$$Ni_{b_1} - N_L i_L = (H_1 - H_L)\ell \tag{215}$$

And, from Fig. 17,

$$Ni_{b_2} - N_L i_L = (H_2 - H_L)\ell \tag{216}$$

$$\cdots\cdots\cdots\cdots$$

$$Ni_{b_n} - N_L i_L = (H_n - H_L)\ell$$

From equations (195), (196), and (197),

$$i_{b_1} = \ldots = i_{b_n} = i_b = I_{bo} + i_p \tag{217}$$

And from equations (174) and (196),

$$i_L = \frac{N}{N_L} i_p \tag{218}$$

Substituting equations (217) and (218) into (215),

$$N(I_{bo} + i_p) - N_L(\frac{N}{N_L} i_p) = (H_1 - H_L)\ell \tag{219}$$

Or

$$NI_{bo} = (H_1 - H_L)\ell \tag{220}$$

By the same manner,

$$\begin{aligned} NI_{bo} &= (H_2 - H_L)\ell \\ &\cdots\cdots\cdots \\ NI_{bo} &= (H_n - H_L)\ell \end{aligned} \tag{221}$$

From equations (220) and (221),

$$H_1 = H_2 = \ldots = H_n \tag{222}$$

Call the value of the individual terms of equations (222) H. Then

$$NI_{bo} = (H - H_L)\ell \tag{223}$$

Assuming no leakage flux, the sum of the flux into the top node is

$$\phi_1 + \phi_2 + \ldots + \phi_n + \phi_L = 0 \tag{224}$$

Since each leg of the transformer is constructed of the same material, then each leg has the same magnetization curve. Or, for each leg, the relationship between the flux density B and the magnetic intensity H is the same. Thus, considering equations (222),

$$B_1 = B_2 = \ldots = B_n \tag{225}$$

Call the value of the individual terms of equations (225) B.

For any magnetic circuit,

$$\phi = \int_S \overline{B} \cdot d\overline{a} \tag{226}$$

For the magnetic circuit of Fig. 17, the flux density B is assumed to be normal to the cross-sectional area of the legs, and to be uniformly distributed over the cross-sectional area. Then equation (226) becomes

$$\phi = \int_S \overline{B} \cdot d\overline{a} = BA \tag{227}$$

where A is the cross-sectional area of the legs. Thus, from equations (225) and (227),

$$\phi_1 = \phi_2 = \ldots = \phi_n \tag{228}$$

Call the value of the individual terms of equations (228) ϕ. Substituting equations (228) into (224),

$$n\phi + \phi_L = 0 \tag{229}$$

Or

$$nB + B_L = 0 \tag{230}$$

Equations (223) and (230) describe the magnetic conditions of the core of Fig. 17. Equation (230) states that the flux density B_L, and thus the magnetic intensity H_L, is in the opposite direction from that assumed. The actual magnetization of the core is found by using these equations in conjunction with the magnetization curve of the core material. The solution of these equations is by trial and error. That is, with NI_{bo} known, a value for H is assumed. This value of H will give a value of B from the magnetization curve. A sample magnetization curve is shown in Fig. 18. This assumed value of H and B will give values of H_L and B_L, from equations (223) and (230). The value of B_L from the equation is then checked with the value of B_L from the magnetization curve. When these two values of B_L are equal, then the assumed values of H and B and the calculated values of H_L and B_L are correct.

CHAPTER VII

EXPERIMENTAL RESULTS

It is felt that the proper experiments that should have been performed for this study are those which would allow comparison of the push-pull series magnetic circuit and the common-input series magnetic circuit with the push-pull parallel magnetic circuit and the common-input parallel magnetic circuit. However, proper facilities and adequate time were not available to permit careful construction of a well-designed output transformer with a parallel magnetic circuit. Thus it is felt that a fair comparison could not be obtained between the circuits using an output transformer for the latter two circuits which is neither properly designed nor properly constructed. The experiments performed were those which will determine the operation of the push-pull parallel magnetic circuit and the common-input parallel magnetic circuit having a transformer of three parallel branches.

The transformer for the push-pull parallel magnetic circuit and the common-input parallel magnetic circuit was constructed on a core taken from a television power transformer. The construction and dimensions of the transformer are shown in Fig. 19.

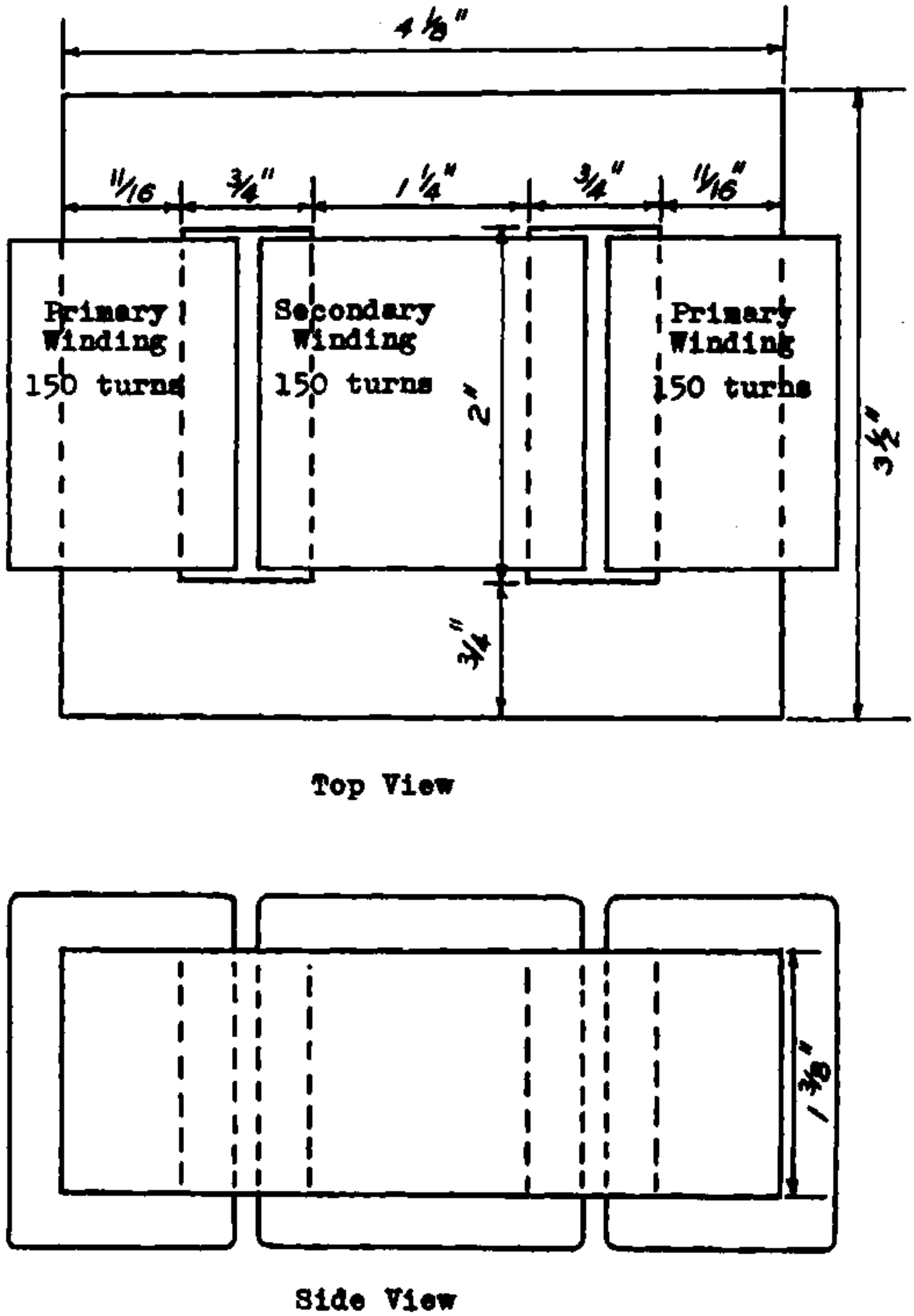


Fig. 19. Transformer Used in Experiments.

The push-pull parallel magnetic circuit of two tubes was assembled using a 6SN7 vacuum tube. The circuit connection is shown in Fig. 20. The observed waveforms are shown in Fig. 22 for a sinusoidal input to the grids. It will be noted from the waveform of the vacuum tube output voltage e_p that the tubes were operating in the non-linear regions of their characteristics. The load voltage e_L was observed to be a sine wave of fundamental frequency free from any noticeable distortion. According to the theory developed, the power supply current should be free from any a-c component. However, a second harmonic component was observed to be present in the power supply current. It is thought that this component could be substantially reduced by use of a properly designed output transformer.

The common-input parallel magnetic circuit of two tubes was assembled using a 6SN7 vacuum tube. The circuit connection is shown in Fig. 21. The observed waveforms are shown in Fig. 23 for a sinusoidal input to the grids. It will be noted that the tubes were operating in the non-linear regions of their characteristics. The output voltages of the tubes and the load voltage had the same waveform, because there was no cancellation of harmonics in the output. The plate currents of the tubes and the power supply current had the same waveform, because there was no cancellation of harmonics in the power supply.

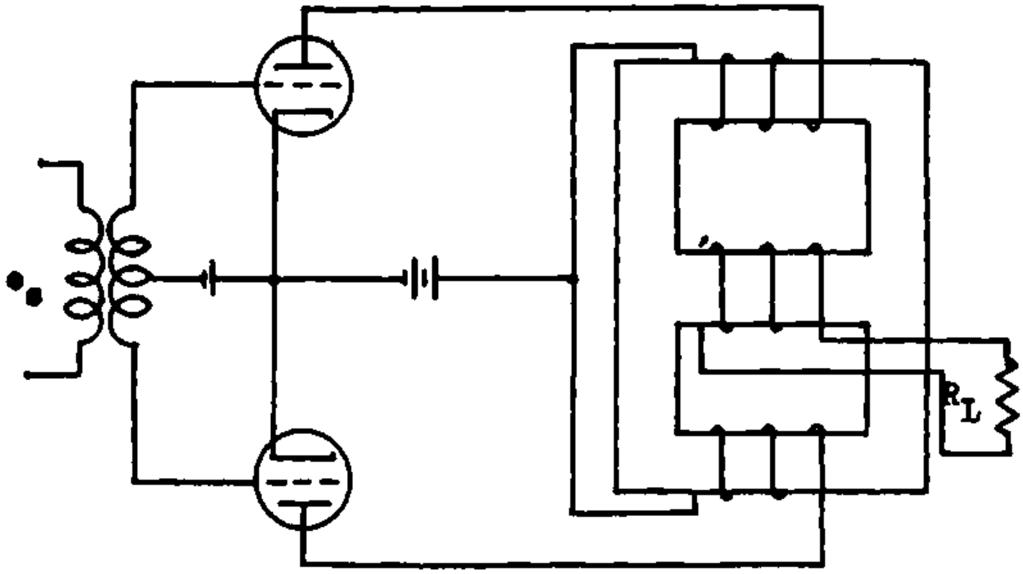


Fig. 20. Two-Tube Push-Pull Parallel Magnetic Circuit.

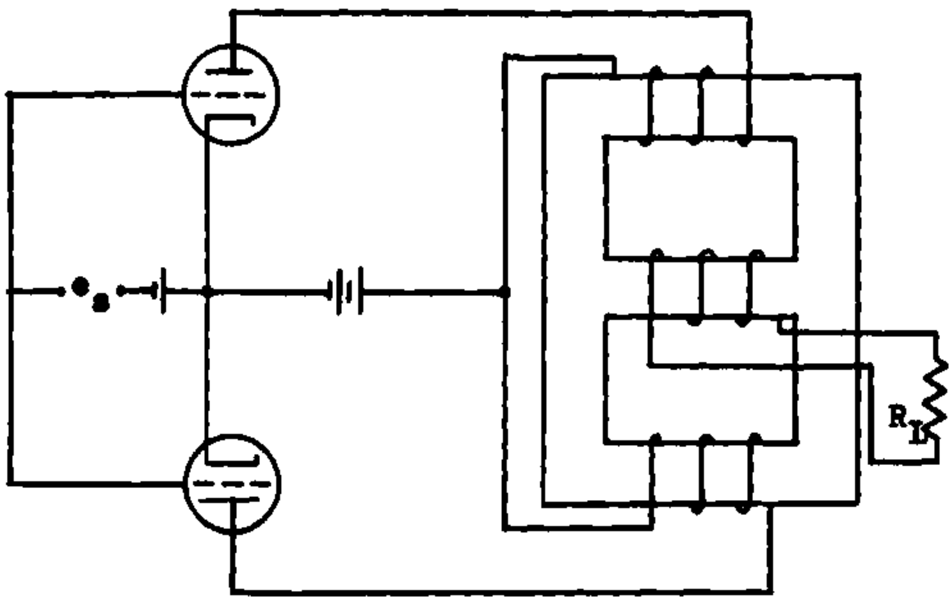
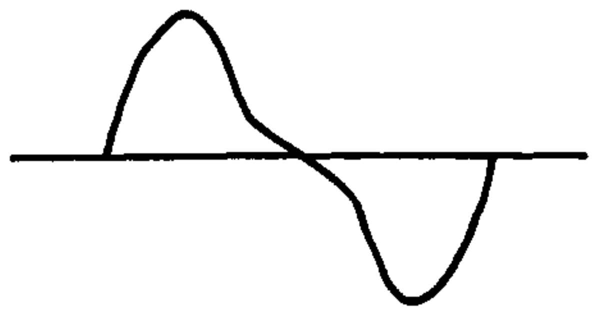
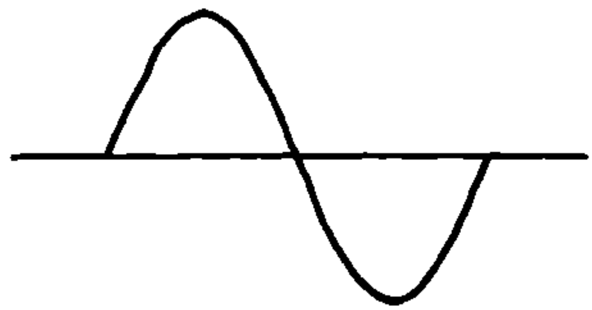


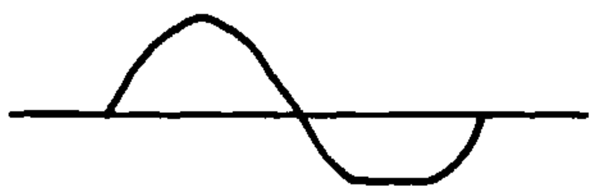
Fig. 21. Two-Tube Common-Input Parallel Magnetic Circuit.



(a) Tube Output Voltages e_p and e_p'.



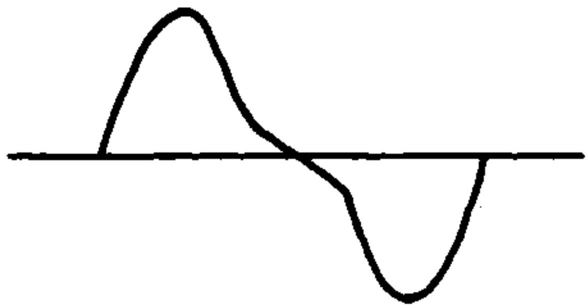
(b) Load Voltage e_L.



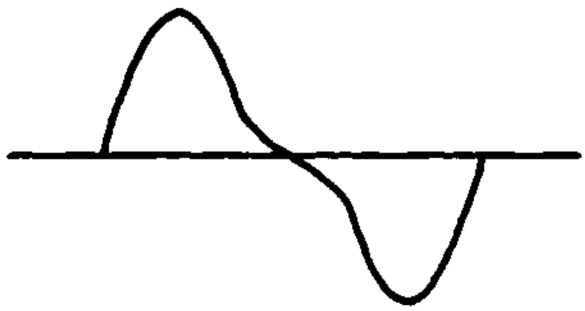
(c) Plate Currents i_p and i_p'.

(d) Power Supply Current.

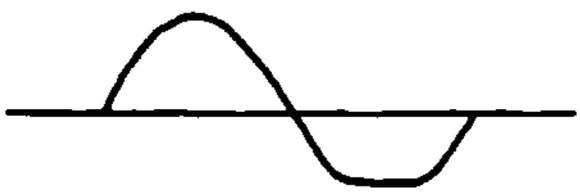
Fig. 22. Observed Waveforms of the Push-Pull Parallel Magnetic Circuit.



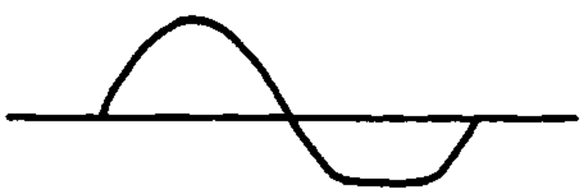
(a) Tube Output Voltages e_p.



(b) Load Voltage e_L.



(c) Plates Currents i_p.



(d) Power Supply Current.

Fig. 23. Observed Waveforms of the Common-Input Parallel Magnetic Circuit.

CHAPTER VIII

CONCLUSIONS

It will be noted that the electrical circuits of the push-pull series magnetic circuit and the push-pull parallel magnetic circuit are the same. The difference in the two circuits is in the topology of the magnetic circuits of the output transformers.

It will also be observed that the electrical circuits of the common-input series magnetic circuit and the common-input parallel magnetic circuit are the same. The difference in the two circuits is in the topology of the magnetic circuits of the output transformers.

In addition, it will be noted that the electrical circuits of all four circuits are the same, except for the manner of applying the grid-signal voltages.

The advantages of the push-pull series magnetic circuit are the same as those of the conventional push-pull circuit. These advantages are increased power output for a specified allowable distortion, no d-c magnetization of the core, reduced sensitivity to ripple voltage in the power supply, and cancellation of odd harmonics in the power supply current.

The advantages of the push-pull parallel magnetic circuit are increased power output for a specified allowable distortion, reduced sensitivity to ripple voltage in the power supply, and cancellation of all harmonics in the power supply current. There is d-c magnetization of the primary legs of the output transformer, but no d-c magnetization of the secondary leg.

There is no cancellation of harmonics in the output of either the common-input series magnetic circuit or the common-input parallel magnetic circuit. D-C magnetization exists in the cores of the output transformers of both circuits. For any given value of load resistance, the output current in the common-input series magnetic circuit is the sum of the plate currents of the vacuum tubes, neglecting the turns ratio. For the common-input parallel magnetic circuit, the output voltage is the sum of the output voltages of the vacuum tubes, neglecting the turns ratio.

For the push-pull series magnetic circuit and the common-input series magnetic circuit, the effect of increasing the number of tubes is to increase the value of load resistance as seen by each tube. For the push-pull parallel magnetic circuit and the common-input parallel magnetic circuit, the effect of increasing the number of tubes is to decrease the value of load resistance as seen by each tube.

Nothing could be found in available literature to indicate that the push-pull circuit using the parallel magnetic-path transformer has ever been investigated. The theory developed in this thesis indicates that the fundamental and all harmonics of current in the power supply are suppressed. Experiments confirmed that there is a low alternating current content in the power supply current. It is felt that a push-pull circuit using a well-designed parallel magnetic-path transformer might be useful in applications where feedback between stages through the power supply is undesirable. The theory developed as a result of this investigation shows no outstanding advantages in either of the common-input circuits.

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